THE CALCULATION OF THE ERGODIC PROJECTION FOR MARKOV CHAINS AND PROCESSES WITH A COUNTABLE INFINITY OF STATES

BY

DAVID G. KENDALL and G. E. H. REUTER

Magdalen College
Oxford

Victoria University
Manchester

1. Introduction

1.1. Let $P \equiv \{p_{ij} : i, j = 0, 1, 2, \ldots\}$ be the matrix of one-step transition probabilities for a temporally homogeneous Markov chain with a countably infinite set of states (labelled as $0, 1, 2, \ldots$). The probability p_{ij}^n of a transition in n steps from state i to state j will then be the (i, j)th element of the matrix P^n, so that the specification of P (or equivalently of $\Delta \equiv P - I$) completely determines the system. It is known (Kolmogorov [24]) that the Cesàro limits

$$\pi_{ij} \equiv \lim_{n\to\infty} \frac{1}{n} \sum_{r=1}^{n} p_{ij}^r \tag{1}$$

always exist. Let Π denote the matrix whose (i, j)th element is π_{ij}, and consider

Problem A: *Determine* Π *when* P *is given.*

This problem has obvious importance for practical applications. A number of special techniques are available for its solution in particular cases (see, e.g., Feller [12], Ch. 15, Foster [14], [15] and Jensen [18]); also Feller [12], pp. 332–4, has given a general iterative method of solution. We shall give another (non-iterative) general method in § 2: it will involve the non-negative solutions of

$$x_j = \sum_{\alpha} x_\alpha \, p_{\alpha j}$$

such that $\sum x_\alpha < \infty$, and the non-negative solutions of

$$y_i = \sum_{\alpha} p_{i\alpha} \, y_\alpha$$

such that $\sup\limits_{\alpha} y_\alpha < \infty$.