DENUMERABLE MARKOV PROCESSES AND THE ASSOCIATED CONTRACTION SEMIGROUPS ON l

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§ 1. Introduction and summary

1.1. We shall be concerned with the analytical rather than with the probabilistic side of the theory of Markov processes. It will therefore be appropriate to define a *process* as a set $\mathcal{P} \equiv \{p_{ij}\}$ of real-valued functions defined on $\langle 0, \infty \rangle$, where i and j range over some fixed denumerable set E, and

$$\text{I}: \qquad p_{ij}(t) \geq 0 \qquad (t \geq 0);$$

$$\text{II}: \qquad \sum_{\alpha \in E} p_{i\alpha}(t) \leq 1 \qquad (t \geq 0);$$

$$\text{III}: \qquad p_{ij}(u+v) = \sum_{\alpha \in E} p_{i\alpha}(u)\, p_{\alpha j}(v) \qquad (u \geq 0,\ v \geq 0);$$

$$\text{IV}: \qquad p_{ij}(0) = \delta_{ij} = \lim_{t \downarrow 0} p_{ij}(t).$$

The continuity condition IV is designed to exclude excessively irregular behaviour (such as non-measurability) of the p_{ij}; it implies the continuity, uniform for $t \geq 0$, of each p_{ij} (Kendall [16], Th. 3.3).

In the probabilistic theory[1] II is strengthened to

$$\text{II}^*: \qquad \sum_{\alpha \in E} p_{i\alpha}(t) = 1.$$

The $p_{ij}(t)$ are then transition probabilities for a time-homogeneous Markov process with E as its set of states. When the sign of inequality is allowed in II, a probabilistic interpretation is still possible if we suppose that E does not exhaust the set

[1] See Doob [4], [5], Ch. VI.