

SUBGROUPS OF IA AUTOMORPHISMS OF A FREE GROUP

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1. Introduction

Generators and defining relations for the group A_n of automorphisms of a free group of rank n were derived by J. Nielsen [11]. For $n=2$, this is a fairly easy task, but for $n \geq 3$ it requires very difficult combinatorial arguments which have not been simplified since the appearance of Nielsen's paper. In order to obtain an easier approach to the investigation of A_n and a better understanding of its structure, it seems natural to study its subgroups.

For all n , the elements of A_n which induce the identical automorphism in the commutator quotient group F_n/F'_n form a normal subgroup K of A_n . Bachmuth [1] calls this the group of IA automorphisms of F_n . Magnus [8] showed that this subgroup is generated by the automorphisms

$$K_{ij}: a_i \rightarrow a_j a_i a_j^{-1}$$

$$a_k \rightarrow a_k, \quad k \neq i$$

and

$$K_{ijk}: a_i \rightarrow a_i a_j a_k a_j^{-1} a_k^{-1}$$

$$a_m \rightarrow a_m, \quad m \neq i$$

where a_1, a_2, \dots, a_n are a set of free generators of F_n , and where the subscripts of each of these automorphisms are distinct members of the set $\{1, 2, \dots, n\}$. In the present paper, we will study certain interesting subgroups of K , in the case $n=3$. In this case, K has a minimal set of nine generators, as K_{ijk}^{-1} is easily seen to be K_{ikj} . Some, although not all, of our results can be obtained for $n > 3$ by the same methods.

In section 3, generators and defining relations for the subgroup K_1 of those automorphisms in K which keep two generators of the free group fixed will be presented. In section 4, generators for the subgroup \tilde{K}_3 of those automorphisms in K which leave one generator of the free group fixed will be found. Then, in section 5, the group of those automorphisms