THE INHOMOGENEOUS MINIMUM OF A TERNARY QUADRATIC FORM (II)

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1. Let $Q(x, y, z)$ be an indefinite ternary quadratic form with real coefficients and determinant $D \neq 0$. For any real numbers x_0, y_0, z_0 we write

$$M(Q; x_0, y_0, z_0) = \text{g.l.b.}\ |Q(x, y, z)|, \tag{1.1}$$

where the lower bound is taken over all sets

$$x, y, z \equiv x_0, y_0, z_0 \pmod 1.$$

Then
$$M(Q) = \text{l.u.b.}\ M(Q; x_0, y_0, z_0), \tag{1.2}$$

over all sets x_0, y_0, z_0, is called the *inhomogeneous minimum* of Q.

In a recent paper [1] I showed that

$$M(Q) < (\tfrac{4}{15})\,|D|)^{\frac{1}{3}} \tag{1.3}$$

unless Q is equivalent to a multiple of one of

$$Q_1 = x^2 - y^2 - z^2 + xy - 7yz + zx \tag{1.4}$$

or
$$Q_2 = 2x^2 - y^2 + 15z^2; \tag{1.5}$$

while
$$M(Q_1) = (\tfrac{27}{100}|D|)^{\frac{1}{3}}, \qquad M(Q_2) = (\tfrac{4}{15}|D|)^{\frac{1}{3}}, \tag{1.6}$$

the upper bound (1.2) being attained only when $x_0, y_0, z_0 \equiv \tfrac{1}{2}, \tfrac{1}{2}, \tfrac{1}{2} \pmod 1$.

These results were an extension of those given by Davenport [5], who showed that there existed a constant $\delta > 0$ such that

$$M(Q) \leq (1 - \delta)\,(\tfrac{27}{100}|D|)^{\frac{1}{3}}$$