On the existence of special metrics in complex geometry

by

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Dedicated to Marcel Friedmann on the occasion of his 80th birthday

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0. Introduction

Every complex manifold X admits a smooth hermitian metric and, in fact, a huge space of them. To study the manifold it is often useful to pick from this collection a particularly nice metric. For example, if X is algebraic, one can always choose the metric to be Kählerian, and often to be Kähler-Einstein (cf. Yau [15]).

The topological and analytic consequences of the existence of a Kähler metric are strong and have been well understood for some time. Recently a characterization of which complex manifolds admit such metrics has been given [8]. However, relatively little is known about how to choose a good metric in general, and this paper represents a first step towards understanding this question. We shall here define and characterize a class of complex manifolds which admit a special type of hermitian metric. This class contains the Kähler manifolds as well as many important categories of non-Kähler manifolds, including, for example: 1-dimensional families of Kähler varieties, the "twistor spaces" constructed from self-dual riemannian 4-manifolds, and complex solvmanifolds. We shall carry out an extensive analysis of this class of manifolds. It is hoped that our results will be important in the further study which now seems clearly worthwhile.