The elementary theory of large e-fold ordered fields

by

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Introduction

The aim of this work is to continue Van den Dries' treatment of the theory of e-fold ordered fields along the lines of the treatment given in Jarden-Kiehne [10] to the theory of e-free Ax fields.

Van den Dries generalizes in his thesis [18] the theory of real closed fields. He considers structures $(K, P_1, \ldots, P_e)$ that consist of a field K and e orderings $P_1, \ldots, P_e$ of K. He proves in [18, p. 54]:

The theory of e-fold ordered fields OF_e has a model companion $\overline{OF}_e$, the models of which are the e-fold ordered fields $(E, P_1, \ldots, P_e)$ satisfying:

(α) P_i and P_j induce different order topologies on E for all $1 \leq i < j \leq e$.

(β) If $f \in E[T, X]$ is an irreducible polynomial and if there exists an $a_0 \in E$ such that $f(a_0, X)$ changes sign on E with respect to each of the P_i's, then there exist $a, b \in E$ such that $f(a, b) = 0$.

In particular it follows from this theorem that the absolute Galois group $G(E)$ of E is a pro-2-group generated by e involutions (cf. [18, p. 77 and p. 92]). If E is algebraic

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