

# The elementary theory of large $e$ -fold ordered fields

by

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## Introduction

The aim of this work is to continue Van den Dries' treatment of the theory of  $e$ -fold ordered fields along the lines of the treatment given in Jarden-Kiehne [10] to the theory of  $e$ -free  $Ax$  fields.

Van den Dries generalizes in his thesis [18] the theory of real closed fields. He considers structures  $(K, P_1, \dots, P_e)$  that consist of a field  $K$  and  $e$  orderings  $P_1, \dots, P_e$  of  $K$ . He proves in [18, p. 54]:

The theory of  $e$ -fold ordered fields  $OF_e$  has a model companion  $\overline{OF}_e$ , the models of which are the  $e$ -fold ordered fields  $(E, P_1, \dots, P_e)$  satisfying:

( $\alpha$ )  $P_i$  and  $P_j$  induce different order topologies on  $E$  for all  $1 \leq i < j \leq e$ .

( $\beta$ ) If  $f \in E[T, X]$  is an irreducible polynomial and if there exists an  $a_0 \in E$  such that  $f(a_0, X)$  changes sign on  $E$  with respect to each of the  $P_i$ 's, then there exist  $a, b \in E$  such that  $f(a, b) = 0$ .

In particular it follows from this theorem that the absolute Galois group  $G(E)$  of  $E$  is a pro-2-group generated by  $e$  involutions (cf. [18, p. 77 and p. 92]). If  $E$  is algebraic

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