Disjoint spheres, approximation by imaginary quadratic numbers, and the logarithm law for geodesics

by

DENNIS SULLIVAN

I.H.E.S., Bures-sur-Yvette, France

Contents

§ 0. Introduction

This paper is based on the principle that probabalistic independence of certain sets in Euclidean space is forced by a disjoint collection of spheres in a Euclidean space of one higher dimension. (See Figure 1.)

This principle allows a new proof of (a new variant of) Khintchine's approximation theorem for almost all reals by rationals § 3. The new proof extends naturally to the approximation of almost all complex numbers by ratios of integers $p/q, p, q \in \vartheta(\sqrt{-d})$ in imaginary quadratic fields.

Let $0 \leq a(x) < 1, x$ a positive real, be any function so that the size of $a(x)$ up to bounded ratio only depends on the size of x up to bounded ratio. The following theorem is proved in § 7.