# Embedding $l_p^m$ into $l_1^n$

by

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## 1. Introduction

It is by now well-known that for any $1<s<2$, $L_1(0,1)$ contains a subspace which is isometrically isomorphic to $l_s$. This of course implies that for any $m=1,2,\ldots$ and any $\varepsilon>0$, $l_s^m$ is $1+\varepsilon$-isomorphic to a subspace of $l_1^n$ if $n=n(\varepsilon,s,m)$ is sufficiently large. Theorem 1, the result of this paper, states that $n$ can be of order $m$; i.e., that $n$ can be chosen smaller than $\beta^{-1}m$ for some constant $\beta=\beta(\varepsilon,s)>0$. This complements the theorem of Figiel, Lindenstrauss and Milman [4] (cf. also [2], [3] for a somewhat weaker result) which treated the case $s=2$.

Actually the proof of Theorem 1 yields more than the above-mentioned result. First, it shows for $0<s<2$ and $0<r<s$ with $r\leqslant 1$, that for every $\varepsilon>0$, $l_s^m$ is $1+\varepsilon$-isomorphic to a subspace of $l_r^n$ if $m\leqslant\beta n$, where $\beta=\beta(\varepsilon,s,r)>0$ is a constant independent of $n$. Secondly, the condition that the range of the isomorphism be $l_r^n$ can be relaxed. What is needed is that the range be an $r$-normed space which possesses a basis $(e_i)_{i=1}^n$ so that for all scalars $(b_i)_{i=1}^n$,

$$\operatorname*{Av}_{\pm}\left\|\sum_{i=1}^{n}\pm b_i e_i\right\|\approx\left(\sum_{i=1}^{n}|b_i|^r\right)^{1/r}.$$

The proof of Theorem 1, like the earlier proof of the $s=2$ case in [4], [2], [3], [5] and [11], is probabilistic in nature. A schematic outline of the usual argument specialized to the case $1<s<2$ and $r=1$ goes like this: For appropriate $m$ and $n$, one defines a probability space $(\Omega,P)$ and a random linear operator or matrix $A=A_\omega$ $(\omega\in\Omega)$ from $l_s^m$

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