Isometry groups of simply connected manifolds of nonpositive curvature II

by

PATRICK EBERLEIN([1])

University of North Carolina
Chapel Hill, N.C., U.S.A.

Introduction

Let H denote a complete simply connected Riemannian manifold of nonpositive sectional curvature, and let $I(H)$ denote the group of isometries of H. In this paper we consider density properties of subgroups $D \subseteq I(H)$ that satisfy the *duality condition* (defined below). These density properties also yield characterizations of Riemannian symmetric spaces of noncompact type and results about lattices in H that strengthen several of the results of [11] and [15]. If H is a symmetric space of noncompact type and if D is a subgroup of $I_0(H)$, then the duality condition for D is implied by the Selberg property (S) for D [20, pp. 4–6] or [10]. A partial converse is obtained in [10]. It is an interesting question whether the two conditions are equivalent in this context.

Our density results are very similar to those of [5]. In Proposition 4.2 we obtain a differential geometric version of the Borel density theorem (cf. Corollary 4.2 of [5]): Let H admit no Euclidean de Rham factor, and let $G \subseteq I(H)$ be a subgroup whose normalizer D in $I(H)$ satisfies the duality condition. Then either (1) G is discrete or (2) there exist manifolds H_1, H_2 such that (a) H is isometric to the Riemannian product $H_1 \times H_2$, (b) H_1 is a symmetric space of noncompact type, (c) $(\bar{G})_0 = I_0(H_1)$ and (d) there exists a discrete subgroup $B \subseteq I(H_2)$, whose normalizer in $I(H_2)$ satisfies the duality condition, such that $I_0(H_1) \times B$ is a subgroup of $\bar{G}$ of finite index in $\bar{G}$. Using the result just quoted or the main theorem of section 3 we then obtain the following decomposition of a manifold H whose isometry group $I(H)$ satisfies the duality condition (Proposition 4.1): Let $I(H)$ satisfy the duality condition. Then there exist manifolds H_0, H_1 and H_2, two of which may have dimension zero, such that (1) H is isometric to

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