GROUPS OF CONTINUOUS FUNCTIONS IN HARMONIC ANALYSIS

BY

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0. Introduction

Let G be a general abelian topological group. We shall denote by $\hat{G}$ its character group, i.e. the multiplicative group of all continuous homomorphisms

$$\chi: G \to \mathbf{T}$$

of G into $\mathbf{T}=\mathbf{R}$ (mod 2π). For every $g \in G$ and every $\chi \in \hat{G}$ we shall denote $\langle \chi, g\rangle = \chi(g)$.

Let $K \subset G$ be a subset of G, we shall say that K is independent if:

$$n_j \in \mathbf{Z}, \quad k_j \in K, \quad j=1,2,\dots,p; \quad \sum_{j=1}^{p} n_j k_j = 0_G \;\Rightarrow\; n_j = 0, \quad j=1,2,\dots,p.$$

(0_G is of course the zero element of G).

Let $E \subset G$ be a subset of G; we shall denote by Gp (E) the subgroup algebraically generated in G by E. Let $K \subset G$ be a compact subset of G, we shall say that K is a Kronecker set if for every $f \in \mathbf{C}(K)$ such that $|f| \equiv 1$ and every $\varepsilon > 0$ we can find some $\chi \in \hat{G}$ such that:

$$\sup_{k \in K} |f(k) - \chi(k)| \leqslant \varepsilon.$$

Let $K \subset G$ be a compact subset of G, we shall say that K is an H_α set for some $\alpha \in (0, 1]$ if for every $f \in \mathbf{C}(K)$ and every $\varepsilon > 0$ we can find a sequence of characters $\{\chi_n \in \hat{G}\}_{n=1}^{\infty}$ and a sequence of complex numbers $\{\alpha_n \in \mathbf{C}\}_{n=1}^{\infty}$ such that:

$$f(k) = \sum_{j=1}^{\infty} \alpha_j \chi_j(k),\ k \in K; \qquad \sum_{j=1}^{\infty} |\alpha_j| \leqslant \alpha^{-1} \|f\|_\infty + \varepsilon.$$

We shall say that a compact set $K \subset G$ is a set of interpolation if it is an H_α set for some $\alpha \in (0, 1]$. We refer the reader to [1] and [2] for elaborations of the above definitions.