# ARITHMETIC MEANS AND THE TAUBERIAN CONSTANT .474541.

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## 1. Introduction.

Let $\Sigma u_n$ be a series of complex terms satisfying the Tauberian condition $\limsup |nu_n| < \infty$. Let $s_n = u_0 + u_1 + \cdots + u_n$ denote the sequence of partial sums of $\Sigma u_n$, and let

$$M_n = \frac{s_0 + s_1 + \cdots + s_n}{n+1} = \sum_{k=0}^{n} \left(1 - \frac{k}{n+1}\right) u_k \tag{1.1}$$

denote the arithmetic mean transform. The Kronecker formula

$$M_n - s_n = \frac{1}{n+1} \sum_{k=0}^{n} k u_k, \tag{1.2}$$

which follows from (1.1), implies that the formula

$$\limsup_{n\to\infty} |M_n - s_{p_n}| \leqq B \limsup |n u_n| \tag{1.3}$$

holds when $p_n = n$ and $B = 1$.

The questions with which we are concerned are the following where in one case we assume that $\Sigma u_n$ has bounded partial sums, and in the other case we do not make this assumption. How much can we reduce the constant $B$ in (1.3) if, instead of requiring that $p_n = n$, we allow $p_n$ to be the optimum sequence that can be selected after the series $\Sigma u_n$ has been given? It was shown in [3, Theorem 5.4] that $B$ can be reduced to $\log 2 = .69315$, and no further, if we require that $p_n$ be a function of $n$ alone so that $p_n$ must be independent of the terms of $\Sigma u_n$. Moreover (1.3) holds when $p_n = [n/2]$ and $B = .69315$. It was also shown in [3, Theorem 9.2] that $B$ can be reduced to .56348 by choosing $p_n$ to be the most favorable one of the two integers $[3n/8]$ and $[5n/8]$, the choice being allowed to depend upon the