SETS OF UNIQUENESS FOR FUNCTIONS REGULAR IN THE UNIT CIRCLE.

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1. For a large number of classes C of functions $f(z)$ regular in the unit circle, we have very complete knowledge concerning the existence of a boundary function

$$F(\theta) = \lim_{r \to 1} f(r e^{i\theta}),$$

the classical result being that of Fatou. However, very little is known about the properties of this boundary function $F(\theta)$, and in particular about the sets E associated with the class C, having the property that $f(z)$ vanishes identically if $F(\theta) = 0$ on E. Let us call a closed set of this kind a *set of uniqueness* for the class C. If E is not a set of uniqueness, we speak of a set of multiplicity. Our whole knowledge in this direction seems to be contained in a classical result of F. and M. Riesz: E is a set of uniqueness for the class of bounded functions if and only if it has positive Lebesgue measure.

We shall here consider the problem of finding the sets of uniqueness for three different classes of functions, namely:

1°. Functions with high regularity in $|z| \leq 1$;

2°. Functions with a bounded Dirichlet integral;

3°. Absolutely convergent Taylor series.

The first class gives us information regarding the nature of the boundary function of analytic functions in general and shows clearly the decisive role of the integral

$$\int_0^{2\pi} \log |f(r e^{i\theta})| \, d\theta.$$