AN EXTENSION OF THE SLUTZKY-FRÉCHET THEOREM.

By

J. M. HAMMERSLEY.

Lectureship in the Design and Analysis of Scientific Experiment, University of Oxford.

§ 0. Notation and conventions.

In this paper German capital letters denote Euclidean vector spaces of finite dimensionality. Small German letters denote point sets in these spaces; and $\mathfrak{x} - \mathfrak{x}'$ denotes the (perhaps empty) set of all points which belong to $\mathfrak{x}$ and not to $\mathfrak{x}'$. Script letters denote classes of point sets. Clarendon type denotes points (or vectors) of a Euclidean space. Ordinary italic type is reserved for scalar quantities. The symbol $\Rightarrow$ denotes implication, the arrow pointing from the premiss to the conclusion; and the double-headed arrow $\Leftrightarrow$ means 'implies and is implied by'. Two statements I and II, which together imply a third III, are linked by an ampersand: — 'I & II $\Rightarrow$ III'.

§ 1. Introduction.

Let $y(x)$ be a continuous one-valued function of x, and consider the equations

$$\lim_{\nu\to\infty} x_\nu = x, \tag{1.1}$$

$$\lim_{\nu\to\infty} (x_\nu - x) = 0, \tag{1.2}$$

$$\lim_{\nu\to\infty} \{y(x_\nu) - y(x)\} = 0, \tag{1.3}$$

$$\lim_{\nu\to\infty} y(x_\nu) = y(x). \tag{1.4}$$

When x and x_ν are real variables, it is familiar that

$$(1.1) \Leftrightarrow (1.2) \Rightarrow (1.3) \Leftrightarrow (1.4). \tag{1.5}$$

For random variables, the position is different. Slutzky (**4**) proved

$$(1.2) \Rightarrow (1.3) \tag{1.6}$$

when x_ν is a random variable and x a real variable; while Fréchet (**1**) proved (1.6)