C^*-ALGEBRAS OF ALMOST PERIODIC PSEUDO-DIFFERENTIAL OPERATORS

BY

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0. Introduction

Our basic goal is to develop an index theory for almost periodic pseudo-differential operators on $\mathbf{R}^n$. The prototype of this theory is [5] which has direct application to the almost periodic Toeplitz operators. Here, we study index theory for a C^*-algebra of operators on $\mathbf{R}^n$ which contains most almost periodic pseudo-differential operators such as those arising in the study of elliptic boundary value problems for constant coefficient elliptic operators on a half space with almost periodic boundary conditions.

Our program is as follows: We begin with a discussion of a C^*-algebra with symbol which contains all of the classical pseudo-differential operators on $\mathbf{R}^n$. Precisely, if A is a bounded operator on $L^2(\mathbf{R}^n)$ and $\lambda \in \mathbf{R}^n$, let $\varepsilon_\lambda(A)$ denote the conjugate of A with the function $e^{i\lambda \cdot x}$ acting as a multiplier denoted e_λ. We first study the C^*-algebra of those A for which the function $\lambda \mapsto \varepsilon_\lambda(A)$ has a strongly continuous extension to the radial compactification of $\mathbf{R}^n$. The restriction of this function to the complement of $\mathbf{R}^n$ then gives the usual (principal) symbol $\sigma(A)$ when A is a pseudo-differential operator of order zero (of a suitable type). We characterize the Fourier multipliers in this algebra and the image of the symbol map. We give sufficient conditions for the usual construction of a pseudo-differential operator as well as one of Friedrichs' constructions to give an element of this algebra. In particular, the latter gives a positive linear right inverse for the symbol map—at least when the symbol is sufficiently smooth. In fact, we show in § 3 that the Friedrichs map is a right inverse to the symbol map in the almost periodic case. We expect this to be true in the general case also.

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