THE L^p-INTEGRABILITY OF THE PARTIAL DERIVATIVES OF A QUASICONFORMAL MAPPING

BY

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1. Introduction

Suppose that D is a domain in euclidean n-space R^n, $n \geqslant 2$, and that $f: D \to R^n$ is a homeomorphism into. For each $x \in D$ we set

$$L_f(x) = \limsup_{y \to x} \frac{|f(y) - f(x)|}{|y - x|},$$
$$J_f(x) = \limsup_{r \to 0} \frac{m(f(B(x, r)))}{m(B(x, r))}, \tag{1}$$

where $B(x, r)$ denotes the open n-ball of radius r about x and $m = m_n$ denotes Lebesgue measure in R^n. We call $L_f(x)$ and $J_f(x)$, respectively, the maximum stretching and generalized Jacobian for the homeomorphism f at the point x. These functions are nonnegative and measurable in D, and

$$J_f(x) \leqslant L_f(x)^n \tag{2}$$

for each $x \in D$. Moreover, Lebesgue's theorem implies that

$$\int_E J_f \, dm \leqslant m(f(E)) < \infty \tag{3}$$

for each compact $E \subset D$, and hence that J_f is locally L^1-integrable in D.

Suppose next that the homeomorphism f is K-quasiconformal in D. Then

$$L_f(x)^n \leqslant K J_f(x) \tag{4}$$

a.e. in D, and thus L_f is locally L^n-integrable in D. Bojarski has shown in [1] that a little

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