DECOMPOSITION OF CERTAIN KLEINIAN GROUPS

BY

BERNARD MASKIT

State University of New York, Stony Brook, N.Y. 11790, USA

The purpose of this note is to give an analytic and geometric description of the class of Kleinian groups which are finitely generated and which have an invariant component.

If one starts with a collection of "basic" groups, and forms finite "combinations" of these groups, one gets a class of "constructible" Kleinian groups. In this paper, our combinations occur in the sense of the Combination Theorems appearing in [8] and [9], where the amalgamated subgroups (Combination I) and the conjugated subgroups (Combination II) are trivial or elliptic cyclic or parabolic cyclic.

To describe our basic groups, we recall the following definitions. A point z lies in the *limit set* $\Lambda(G)$ if there is a sequence $\{g_n\}$ of distinct elements of G, and there is a point z_0 with $g_n z_0 \to z$. The *set of discontinuity* $\Omega(G)$ is the complement of $\Lambda(G)$. The connected components of $\Omega(G)$ are called *components* of G. A component Δ_0 of G is *invariant* if $g(\Delta_0) = \Delta_0$ for all $g \in G$.

If $\Lambda(G)$ is a finite set, then G is *elementary*. If G is non-elementary and has a simply-connected invariant component Δ_0, then there is a conformal map φ from Δ_0 onto the unit disc. A parabolic element $g \in G$ is called *accidental* if $\varphi g \varphi^{-1}$ is hyperbolic. By definition, elementary groups do not contain accidental parabolic transformations.

For the purposes of this paper, a *basic group* is a finitely-generated Kleinian group which has a simply-connected invariant component, and which contains no accidental parabolic transformations.

The basic groups are, in a sense, all known, it was shown in [11] (for proof, see Bers [4] and Kra–Maskit [7]) that every basic group is either elementary, degenerate, or quasi-Fuchsian. The *degenerate* groups are such that $\Omega(G)$ is both connected and simply-connected. A *quasi-Fuchsian* group is a quasiconformal deformation of a Fuchsian group.

We define the class $\mathcal{C}_1$ as being the class of Kleinian groups which have an invariant component, and which can be built up in a finite number of steps from the basic groups