HOW A MINIMAL SURFACE LEAVES AN OBSTACLE

BY

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This paper is an investigation of the curve of separation determined by the solution to a variational inequality for minimal surfaces. A strictly convex domain Ω in the $z = x_1 + ix_2$ plane is given together with a smooth function ψ which assumes a positive maximum in Ω and is negative on $\partial\Omega$, the boundary of Ω. Let u denote the Lipschitz function which minimizes area among all Lipschitz functions in Ω constrained to lie above ψ in Ω and to vanish on $\partial\Omega$. For such u there is a coincidence set $I \subset \Omega$ consisting of those points z where $u(z) = \psi(z)$. Let us call $\Gamma = \{(x_1, x_2, x_3): x_3 = u(z) = \psi(z),\ z \in \partial I\}$ the "curve" of separation. The object of this paper is to show that Γ is analytic, as a function of its arc length parameter, provided that ψ is strictly concave and analytic.

The study of the coincidence set of the solution to a variational inequality and its curve of separation was originated, together with the study of the regularity of the solution, by H. Lewy and G. Stampacchia ([11]). They obtained, essentially, the result presented here for the variational inequality derived from the Dirichlet Integral. The topological conclusion that Γ is a Jordan curve was reached under the assumption that $\psi \in C^2(\overline{\Omega})$ be strictly concave, a conclusion valid for a wide variety of cases, in particular the problem treated in this paper ([6]).

Our demonstration relies on the resolution of a system of differential equations and the utilization of the solution to extend analytically a conformal representation of the minimal surface which is the graph of u in the subset of Ω where $u(z) > \psi(z)$. The idea of connecting an analytic function to its possible extension by means of the solution to a differential equation is due to Hans Lewy and was used by him to study the behavior of minimal surfaces with prescribed and with free boundaries ([9], [10]).

The problem at hand is distinguished from more well known problems in the calculus of variations because it has only a single boundary relation, impeding both the derivation

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