THE RESIDUAL LIMIT SETS OF KLEINIAN GROUPS

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The study of Kleinian groups, having venerable roots in the work of Poincaré and Klein, has experienced a resurgence in interest in the period following the discovery of the finiteness theorem by L. Ahlfors. This interest has centered about the geometric and function-theoretic properties of the sets of discontinuity of finitely generated Kleinian groups. Using these recent results and various properties of plane continua, we will discuss the structure of the limit sets of finitely generated Kleinian groups. It is well known that the limit sets of (non-elementary) Kleinian groups are perfect, nowhere dense and have positive capacity. For G a Kleinian group, we denote by $\Lambda_0(G)$, the *residual limit set of G*, which we define to be the set of those limit points of G not lying in the boundary of any component of the set of discontinuity of G. It was classically stated, though incorrect, that $\Lambda_0(G)$ is always void, e.g. in Fricke–Klein ([6] p. 136) we find the following assertion—*Wir sehen, dass die beiden Fixpunkte einer hyperbolischen oder loxodromischen Substitution stets auf einer und derselben Grenzcurve, d. h. auf einem und demselben geschlossenen Zuge der Berandung des Netzes liegen.* The error recurs explicitly in Lehner ([8], p. 108). A counterexample was given by the author in [1]. Indeed, it is shown there that $\Lambda_0(G)$ may have positive areal measure when G is an infinitely generated group.

We are concerned here with the properties of the residual limit set. It is shown that if $\Lambda_0(G)$ is not void, it has a perfect subset. This is related to the zero-measure problem of Ahlfors ([3]). When G is finitely generated, a necessary and sufficient condition for λ to lie in $\Lambda_0(G)$ is the existence of a nested sequence of Jordan curves, each lying in the limit set, converging to λ. As a corollary we find that hyperbolic or loxodromic fixpoints lie in $\Lambda_0(G)$ if and only if the two fixed points of the transformation are separated by a Jordan curve which lies in the limit set. Perhaps most interesting for finitely generated groups is the

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