OPERATORS OF PRINCIPAL TYPE WITH INTERIOR BOUNDARY CONDITIONS

BY

JOHANNES SJÖSTRAND

University of Lund, Lund, Sweden

0. Introduction and statement of the main results

In this paper we shall prove results, extending slightly those announced in [16]. The background is some work of Hörmander [9] and Egorov and Kondratev [5], which we shall first describe briefly. We shall always use the same notations for function spaces as Hörmander [7].

Let Ω be a paracompact C^∞ manifold without boundary, $T^*(\Omega)$ the cotangent space, $T^*(\Omega)\setminus 0$ the space of non zero cotangent vecors and $L^m(\Omega)$ the space of pseudodifferential operators of type 1,0, introduced by Hörmander [8, 10]. In [9] Hörmander studied a pseudodifferential operator $P \in L^m(\Omega)$ with a principal symbol $p \in C^\infty(T^*(\Omega)\setminus 0)$, positively homogeneous of degree m, such that $C_p \neq 0$ everywhere on the set of zeros of p. Here $C_p \in C^\infty(T^*(\Omega)\setminus 0)$ is defined by

$$C_p(x,\xi) = 2\,\mathrm{Im} \sum_{j=1}^{n} p^{(j)}(x,\xi)\,\overline{p_{(j)}(x,\xi)}. \tag{0.1}$$

where $x=(x_1, x_2, ..., x_n)$ are some local coordinates in Ω and $\xi=(\xi_1, \xi_2, ..., \xi_n)$ are the corresponding dual coordinates in the cotangent space and $p^{(j)}=\partial p/\partial \xi_j$ and $p_{(j)}=\partial p/\partial x_j$. If we fix a strictly positive C^∞ density, then the complex adjoint $P^* \in L^m(\Omega)$ (i.e. the adjoint with respect to the corresponding sesquillinear scalar product) is defined and if we write

$$C_p(x,\xi) = -i \sum_{j=1}^{n} \left(p^{(j)}(x,\xi)\,\overline{p_{(j)}(x,\xi)} - \overline{p^{(j)}(x,\xi)}\,p_{(j)}(x,\xi)\right),$$

we see, using the calculus of pseudodifferential operators, that C_p is the homogeneous principal symbol of $[P, P^*]=PP^*-P^*P$. In particular C_p is independent of the choice of local coordinates. The expression

$$\{p, \bar{p}\} = \sum_{1}^{n} \left(p^{(j)}\overline{p_{(j)}} - \overline{p^{(j)}}\,p_{(j)}\right)$$

is known as the Poisson bracket of p and $\bar{p}$.

1 – 732904 *Acta mathematica* 130. Imprimé le 30 Janvier 1973