

EXTENSIONS IN VARIETIES OF GROUPS AND ALGEBRAS

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Introduction

The principal object of the following investigation is to study the *Schreier theory of extensions* and its analogue for any variety V_S of groups or (not necessarily associative) linear algebras defined by a fixed (but arbitrary) set S of identical relations, and then to show how the *Schreier theory* can be applied to give various qualitative results on extensions within such a variety.

Apart from dealing with groups and algebras separately, the discussion treats in a unified way extensions within such varieties as the varieties of (i) all groups, (ii) abelian groups, (iii) groups of fixed exponent k , (iv) groups of fixed nilpotency class k , (v) associative algebras, (vi) commutative and associative algebras, (vii) Lie algebras, and (viii) Jordan algebras, etc; although groups and algebras are treated separately, there are strong analogies between the results obtained for the two cases.