

ON CERTAIN THEOREMS IN OPERATIONAL CALCULUS.

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The object of this paper is twofold: firstly to establish certain theorems in Operational Calculus and secondly to obtain the Laplace transforms of several functions.

I.

1. Let us suppose [1]

$$\Phi(p) = p \int_0^{\infty} e^{-pt} f(t) dt \quad (1)$$

where p is a positive number (or a number whose real part is positive) and the integral on the right converges. We shall then say that $\Phi(p)$ is operationally related to $f(t)$ and symbolically

$$\Phi(p) \doteq f(t) \text{ or } f(t) \doteq \Phi(p). \quad (2)$$

Many interesting relations involving $\Phi(p)$ and $f(t)$ have been obtained. The following will be required in the sequel.

$$p \Phi(p) \doteq \frac{d}{dt} f(t), \text{ if } f(0) = 0 \quad (3)$$

$$p \frac{d}{dp} [\Phi(p)] \doteq -t \frac{d}{dt} f(t) \quad (4)$$

$$\frac{\Phi(p)}{p} \doteq \int_0^t f(t) dt \quad (5)$$

$$p \int_0^{\infty} \frac{\Phi(p)}{p} dp \doteq \frac{f(t)}{t} \quad (6)$$

$$p \frac{d}{dp} \left[\frac{\Phi(p)}{p} \right] \doteq -t f(t). \quad (7)$$