AVERAGES OF THE COUNTING FUNCTION OF A QUASIREGULAR MAPPING

BY

P. MATTILA and S. RICKMAN

University of Helsinki
Helsinki, Finland

1. Introduction

The theory of quasiregular and quasimeromorphic mappings has turned out to form a natural real n-dimensional generalization of the theory of analytic and meromorphic functions of one complex variable. The study of these mappings was initiated by Rešetnjak in 1966 in a series of papers listed in [9]. Since then the theory has been developed in many directions by several authors. For basic parts of it we refer to [9–11]. Definitions are given in 2.1 of Section 2.

Large parts of the theory of analytic functions of one complex variable have their analogs for n-dimensional quasiregular mappings. The methods of proofs for $n \geq 3$ are for the most part completely different from the classical methods in the plane theory. This state of affairs has had its influence also on the classical theory. On one hand, new and sometimes simpler proofs have been found for known theorems. On the other hand, some interesting results are new discoveries for the value distribution theory in the plane.

In this paper we study value distribution of quasiregular mappings in Riemannian manifolds. Let us consider the basic case, a nonconstant quasimeromorphic mapping f of the Euclidean n-space $\mathbf{R}^n$ into $\bar{\mathbf{R}}^n = \mathbf{R}^n \cup \{\infty\}$. The fundamental question of value distribution of f is how $f^{-1}(y)$ is distributed and how this set varies with changing of y. A natural quantitative measurement of the behavior of $f^{-1}(y)$ is the *counting function* $n(r, y)$ which is the number of points of $f^{-1}(y)$ in the ball $|x| \leq r$ with multiplicity regarded. The spherical average $A(r)$ is the average of $n(r, y)$ with respect to the spherical n-measure on $\bar{\mathbf{R}}^n$ when y runs over $\bar{\mathbf{R}}^n$. The well-known covering theorems in Ahlfors's theory of covering surfaces [1, p. 164, 165] imply for $n = 2$ that the average of $n(r, y)$ when y runs over a subdomain or a "regular curve" in $\bar{\mathbf{R}}^2$, is close to $A(r)$ outside a set of radii r with finite logarithmic measure. This suggests that $n(r, y)$ is usually close to $A(r)$ and that "equidistribution" occurs to some