MINIMAL SURFACES WITH FREE BOUNDARIES

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1. Problem and main results

The classical work of R. Courant and H. Lewy has initiated the study of minimal surfaces with free or at least partially free boundaries on prescribed, not necessarily planar surfaces. During the last decade, several authors including S. Hildebrandt, W. Jäger, J. C. C. Nitsche, K. H. Goldhorn, F. P. Harth and J. E. Taylor have investigated the boundary behavior of a minimal surface on its free boundary; see in particular [6], [11], [12], [16], [17], [19]. A survey of the results up to 1975, with an appended bibliography can be found in chapter VI.2, pp. 447–474, and on p. 707 of [18].

Let us consider a typical problem. Given a configuration in Euclidean 3-space $\mathbf{R}^3$ consisting of a smooth 2-dimensional surface $\mathcal{S}$ and of a smooth Jordan arc Γ having its end points P_1 and P_2 on $\mathcal{S}$, but no other points in common with $\mathcal{S}$. We introduce the class $\mathfrak{C} = \mathfrak{C}(\Gamma, \mathcal{S})$ of all surfaces $x = x(w) = (x^1(u, v), x^2(u, v), x^3(u, v))$ in $C^0 \cap H_2^1(B, \mathbf{R}^3)$, $w = u + iv$, which are parametrized over the semi-disc $B = \{w;\ |w| < 1,\ v > 0\}$ and are bounded by the configuration $\langle \Gamma, \mathcal{S} \rangle$ in the following sense:

Denote by C the closed circular arc $\{w;\ |w| = 1,\ v \geq 0\}$ and by I the open interval $\{w;\ |u| < 1,\ v = 0\}$. Moreover, fix a third point P_3 on Γ, different from P_1 and P_2. Let x_C and x_I be the L_2-traces ("boundary values") of $x \in H_2^1(B, \mathbf{R}^3)$ on C and I, respectively. Then, for any surface x in $\mathfrak{C}$ we assume that x_C maps C continuously and in weakly monotonic manner onto Γ such that $x_C(-1) = P_1$, $x_C(1) = P_2$ and $x_C(i) = P_3$, while $x_I(w) \in \mathcal{S}$ almost everywhere on I.

We look for a surface $x(w)$ which minimizes the Dirichlet integral

$$D(x) = \iint_B |\nabla x|^2 \, du \, dv \tag{1.1}$$

in the class $\mathfrak{C}(\Gamma, \mathcal{S})$. It is well known that this variational problem, to be denoted by $\mathcal{D}(\Gamma, \mathcal{S})$, always has at least one solution $x \in \mathfrak{C}$. The position vector x is real analytic in

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