SMALL ZEROS OF ADDITIVES FORMS IN MANY VARIABLES. II(1)

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1. Introduction

It is a well known consequence of the Hardy-Littlewood Circle Method that a diophantine equation

$$a_1x_1^k+\ldots+a_sx_s^k=0 \tag{1.1}$$

has a nontrivial solution in nonnegative integers $x_1, \ldots, x_s$, provided only that $s\geqslant c_1(k)$ and that the coefficients $a_1, \ldots, a_s$ are not all of the same sign. In the first paper [4] under the present title, the author proved that if $\varepsilon>0$, and if at least $c_2(k, \varepsilon)$ of the coefficients are positive and at least $c_2(k, \varepsilon)$ are negative, then the equation has a nontrivial solution in nonnegative integers with

$$|x_i|\leqslant A^{(1/k)+\varepsilon}\quad (i=1, \ldots, s) \tag{1.2}$$

where

$$A=\max(1, |a_1|, \ldots, |a_s|). \tag{1.3}$$

In the equation $b_1(x_1^k+\ldots+x_t^k)-b_2(x_{t+1}^k+\ldots+x_{2t}^k)=0$ where b_1, b_2 are coprime and positive, every nontrivial solution in nonnegative $x_1, \ldots, x_{2t}$ has some $x_i\geqslant(B/t)^{1/k}$ where $B=\max(b_1, b_2)$. This shows that the exponent in (1.2) is essentially best possible.

In particular, it follows that if k is odd, if $s\geqslant 2c_2(k, \varepsilon)$ and if $a_1, \ldots, a_s$ have arbitrary signs, then there is a nontrivial solution of (1.1) in integers $x_1, \ldots, x_s$ (not necessarily nonnegative) with (1.2). This latter result had also been shown by Birch [1]. But much more is true. We will show that if k is odd and if $s\geqslant c_3(k, \varepsilon)$ where $\varepsilon>0$, then (1.1) has a nontrivial solution in integers $x_1, \ldots, x_s$ with

$$|x_i|\leqslant A^{\varepsilon}\quad (i=1, \ldots, s). \tag{1.4}$$

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