LOCALIZATION THEOREM IN K-THEORY FOR SINGULAR VARIETIES ([1])

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0. Introduction

0.1. The preceding paper [1] constructs a map $L.: K_0^{eq}(X) \to K_0^{abs}(|X|) \otimes \Lambda$ for any equivariant quasi-projective X with projective fixed point scheme $|X|$. The construction of $L.$ involves an imbedding in a nonsingular variety; it is proved that $L.$ is independent of the imbedding and is a covariant natural transformation. A fixed point formula results by mapping X to a point. Here we give a direct proof of the following stronger result.

LOCALIZATION THEOREM. *Let i: $|X| \to X$ be the inclusion of the fixed point subvariety. Then the induced map i_* from $K_0^{eq}(|X|) \otimes \Lambda$ to $K_0^{eq}(X) \otimes \Lambda$ is an isomorphism.*

In § 1 we recall the construction of $L.$ for a fixed imbedding of X in a nonsingular variety, and we show that $L. \circ i_*$ is the identity endomorphism of $K_0^{abs}(|X|) \otimes \Lambda$. We prove in § 2 that i_* is surjective. Thus since $L.$ and i_* are inverse isomorphisms, $L.$ is independent of the imbedding. Since i_* is clearly covariant, the covariance of $L.$ follows. One recovers the Lefschetz–Riemann–Roch formula of [1], since the other properties of $L.$ listed in that theorem are consequences of the corresponding properties of i_*.

This localization theorem is an analogue of localization theorems in topological K-theory (cf. [1], 0.8), and Nielsen's result [4] in algebraic K-theory for nonsingular varieties.

0.2. As in [1], "equivariant varieties" are quasi-projective schemes with an endomorphism of finite order prime to the characteristic of the ground field k, such that the fixed point

([1]) Appendix to Lefschetz–Riemann–Roch for singular varieties by P. Baum, W. Fulton and G. Quart.