LEFSCHETZ-RIEMANN-ROCH FOR SINGULAR VARIETIES (1)

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§ 0. Introduction

0.1. The fixed point problem

Let k be an algebraically closed field, and let n be an integer prime to the characteristic of k. By an *equivariant variety* we shall mean a quasi-projective scheme X over k together with an automorphism $x: X \to X$ such that $x^n = \mathrm{id}$. The fixed point scheme will be denoted $|X|$, and its automorphism will be the identity. All morphisms $f: X \to Y$ are assumed to be equivariant, i.e., $y \circ f = f \circ x$, and the induced morphism of fixed point schemes is denoted $|f|: |X| \to |Y|$.

An *equivariant sheaf* on X is a coherent sheaf $\mathfrak{F}$ of O_X modules together with a homomorphism

$$\varphi_{\mathfrak{F}}: x^*\mathfrak{F} \to \mathfrak{F}$$

of sheaves of O_X-modules.

The Lefschetz Fixed Point Problem is to calculate, for an equivariant sheaf $\mathfrak{F}$ on a projective equivariant variety X, the alternating sum of the traces of the induced maps on the cohomology $H^i(X, \mathfrak{F})$, as a sum of contributions from the components of $|X|$. We prove a general Lefschetz–Riemann–Roch theorem which solves the fixed point problem when X is mapped to a point, just as the Hirzebruch–Riemann–Roch formula follows from a general Riemann–Roch theorem [4].

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