RIEMANN-ROCH AND TOPOLOGICAL K-THEORY FOR SINGULAR VARIETIES[1]

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§ 0. Introduction

0.1. Summary

The basic Riemann–Roch problem is to give, for any sheaf S of O_X modules on an algebraic variety X, a formula for $\chi(X, S)$, the alternating sum of the ranks of the sheaf cohomology groups $H^i(X, S)$. Perhaps the most striking fact about $\chi(X, S)$ is that it is constant in a flat family: while the individual ranks of the $H^i(X, S)$ may vary, their alternating sum does not. This invariance under deformation leads one to suspect that $\chi(X, S)$ may be a topological invariant. In this paper we will present the Riemann–Roch Theorem as a transition from algebra to topology; one consequence will be a topological formula for $\chi(X, S)$.

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