

# RIEMANN-ROCH AND TOPOLOGICAL $K$ -THEORY FOR SINGULAR VARIETIES<sup>(1)</sup>

BY

PAUL BAUM, WILLIAM FULTON and ROBERT MACPHERSON

*Brown University, Providence, Rhode Island, U.S.A.*

## Contents

|   |     |
|---|-----|
| § 0. Introduction . . . . .                         | 155 |
| § 1. Relative $K$ -groups . . . . .                 | 160 |
| § 2. Deformation to the normal bundle . . . . .     | 166 |
| § 3. $K$ -cohomology and $K$ -homology . . . . .    | 169 |
| § 4. The Riemann-Roch theorem . . . . .             | 174 |
| § 5. The Chern character . . . . .                  | 179 |
| § 6. Orientations . . . . .                         | 181 |
| Appendix 1. Complexes of vector bundles . . . . .   | 183 |
| Appendix 2. Complexes of sheaves . . . . .          | 185 |
| Appendix 3. The genus of projective space . . . . . | 189 |
| References . . . . .                                | 191 |

## § 0. Introduction

### 0.1. Summary

The basic Riemann-Roch problem is to give, for any sheaf  $\mathcal{S}$  of  $O_X$  modules on an algebraic variety  $X$ , a formula for  $\chi(X, \mathcal{S})$ , the alternating sum of the ranks of the sheaf cohomology groups  $H^i(X, \mathcal{S})$ . Perhaps the most striking fact about  $\chi(X, \mathcal{S})$  is that it is constant in a flat family: while the individual ranks of the  $H^i(X, \mathcal{S})$  may vary, their alternating sum does not. This invariance under deformation leads one to suspect that  $\chi(X, \mathcal{S})$  may be a topological invariant. In this paper we will present the Riemann-Roch Theorem as a transition from algebra to topology; one consequence will be a topological formula for  $\chi(X, \mathcal{S})$ .

---

<sup>(1)</sup> Research partially supported by NSF grants MCS 76-09817 and MCS 76-09753.