ON THE NUMBER OF RESTRICTED PRIME FACTORS OF AN INTEGER. II

BY

KARL K. NORTON(1)

2235 Floral Drive, Boulder, Colorado 80302, U.S.A.

§ 1. Introduction

Let P be the set of all (positive rational) prime numbers, and let E be an arbitrary nonempty subset of P. Throughout this paper, let p denote a general member of P, and for non-negative integers a, write $p^a \| n$ if $p^a | n$ and $p^{a+1} \nmid n$. For each positive integer n, define

$$\omega(n; E) = \sum_{p|n,\, p\in E} 1, \quad \Omega(n; E) = \sum_{p^a\|n,\, p\in E} a.$$

We usually write $\omega(n; P) = \omega(n)$, $\Omega(n; P) = \Omega(n)$. In a previous paper [37], we obtained sharp inequalities for the frequencies of large deviations of $\omega(n; E)$ and $\Omega(n; E)$ from their normal order of magnitude. Those inequalities included refinements of a special case of a general theorem due to Elliott [11, Theorem 6] concerning large deviations of $f(g(n))$, where f is a strongly additive arithmetic function and $g(n)$ is a positive-valued polynomial in n with integral coefficients. Elliott's result was in turn a refinement (under stronger hypotheses) of a theorem of Uždavinis [55]. (The result of Uždavinis is stated as Theorem 3.3 in Kubilius [28].)

The methods used in [37] were "almost" elementary. Here we shall use more difficult methods to obtain asymptotic formulas for large deviations of $\omega(n; E)$ and $\Omega(n; E)$. We shall also generalize some of the results of [37] and give some applications. For a partial survey of the literature in this area, see [39].

In order to state our main theorems, it is necessary to introduce further notation which will be used throughout this paper. First, we define

$$Q(t) = t - (1+t)\log(1+t) \quad \text{for real } t > -1, \tag{1.1}$$
$$Q(-1) = -1 = \lim_{t\to -1+} Q(t).$$

AMS 1970 subject classifications. Primary 10H15, 10H20, 10H25, 10K20.

Key words and phrases. Prime factors, large deviations, divisor functions, additive functions.

(1) Research partially supported by the National Science Foundation under Grant No. MPS75-06657.