ON INVERSE PROBLEMS ASSOCIATED WITH SECOND-ORDER DIFFERENTIAL OPERATORS

BY

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In a by now classical theorem G. Borg [1] proved the following:

THEOREM A. *Consider the two Sturm–Liouville problems*

$$y'' + [\lambda - q(x)]\, y = 0 \tag{1}$$

$$y(0) \cos \alpha + y'(0) \sin \alpha = 0, \quad y(\pi) \cos \beta + y'(\pi) \sin \beta = 0, \tag{2}$$

$$y(0) \cos \alpha + y'(0) \sin \alpha = 0, \quad y(\pi) \cos \gamma + y'(\pi) \sin \gamma = 0, \tag{3}$$

where $q(x)$ *is real and integrable on* $(0, \pi]$ *and sin* $(\gamma - \beta) \neq 0$. *Then the two spectra corresponding to the boundary conditions* (2) *and* (3) *uniquely determine* $q(x)$, *almost everywhere.*

More recently Li [3] proved the following theorem.

THEOREM B. *Consider the boundary value problem*

$$y'' + [\lambda^2 - q(x)]\, y = 0 \tag{4}$$

$$y(0) = 0, \quad a\, y'(\pi) + \lambda\, y(\pi) = 0, \tag{5}$$

where $a \neq 0$ *is real and* $q(x)$ *is integrable on* $[0, \pi]$. *The spectrum of the problem* (4), (5) *uniquely determines* $q(x)$, *almost everywhere.*

At first glance it seems paradoxical that the determination of $q(x)$ depends on two spectra in Theorem A and only one spectrum in Theorem B. It is our purpose to

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