TENSOR ALGEBRAS AND HARMONIC ANALYSIS

BY

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Introduction

The material presented in this paper is a systematic exposition of the theory of tensor algebras and their applications and connections with harmonic analysis.

We shall not attempt here in the introduction to describe or summarize the methods and results of this paper. We shall instead refer the reader to [13], [28] and [29] for that. We shall also refer the reader to [19], [23], [26], and [27] for background reading relative to the two main problems considered in this paper, namely, the problem of "spectral synthesis" and that of "symbolic calculus".

We would like to point out, however, that none of the above literature is an essential pre-requirement for the understanding of this paper. What is needed instead is a certain familiarity with commutative Banach algebras and in particular regular algebras. One can acquire this in [2]; also we shall have to assume in this paper one or two easy but slightly technical points of harmonic analysis that are very well exposed in [5]. Finally some knowledge of the general theory of the topological tensor product as is to be found in [1] is desirable but not essential provided that the instructions given below as to how this paper should be read are carefully followed.

In Ch. 1 we recall definitions and notations from functional analysis and prove some easy lemmas.

In Ch. 2 we define a tensor algebra in two ways: using functional analytic concepts in § 1 and directly in § 2. The reader who wishes to ignore functional analysis should start reading this paper from Ch. 2, § 2.

In Ch. 3 we develop some of the fundamental topological techniques that allow us to work with tensor algebras. There § 4 is the most crucial paragraph and also the easiest to read. The reader can go directly from Ch. 2, § 2 to Ch. 3, § 4 provided that he is prepared to refer back for definitions.