

INEQUALITIES FOR CERTAIN FUCHSIAN GROUPS

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1. Introduction

In one of the earliest papers on automorphic functions, Poincaré constructed functions automorphic with respect to a Fuchsian group by means of the now well known Poincaré series. If G is a Fuchsian group with ∞ an ordinary point of G , the convergence of the Poincaré series depends upon the convergence of the series

$$\sum(G, z, t) = \sum_{v \in G} |V'(z)|^t$$

where z is any ordinary point of G . In 1882, Poincaré [15, p. 206] showed that this series converges if $t > 1$.

Now suppose that G is finitely generated. If G is of the first kind, then [13, p. 181]

$$\sum(G, z, 1) = +\infty \tag{1.1}$$

whereas if G is of the second kind, then [13, p. 178]

$$\sum(G, z, 1) < +\infty. \tag{1.2}$$

An obvious question, then, is to what extent can (1.2) be improved upon. In this paper we show that (1.2) is best possible when regarded as being a statement applicable to all finitely generated Fuchsian groups of the second kind but nevertheless can be improved upon for any given group. More precisely, we prove the following two theorems.

THEOREM I. *Given any number t satisfying $t < 1$, there exists a finitely generated Fuchsian group of the second kind with ∞ an ordinary point of G and with*

$$\sum(G, z, t) = +\infty$$

for every ordinary point z .