THE CURRENTS DEFINED BY ANALYTIC VARIETIES

BY

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Introduction

This paper is concerned with integration on complex analytic spaces and the (De Rham) currents defined by such integration. It contains results about continuity of fibering and intersection of such currents. It also states sufficient conditions for a current to be defined by integration over a complex subvariety.

The methods of proof involve extensive use of the theory relating to the integral currents of H. Federer and W. Fleming, as it is developed in Federer's treatise *Geometric measure theory* [7]. Since this theory is unfamiliar to most complex analysts, we have stated necessary results in Chapter Two and in Section 5.1. Few theorems are needed for the fibering result, but Chapter Five uses the theory in more detail.

Each chapter has a brief introduction, but we will state the main results here.

The fibering theorem of Section 3.3 is stated for holomorphic maps $f: X^m \to Y^n$ between complex analytic spaces X^m and Y^n, where Y^n is locally irreducible and the complex dimension of the fibers $f^{-1}(y)$ equals $m-n$ for all y in Y. If u is a continuous $(2m-2n)$ form with compact support on X^m, u defines a current $[u]$ and for any holomorphic f the current $f_*[u]$ on Y is defined. If f satisfies the hypothesis above, we prove that $f_*[u] = [\lambda]$, the current defined by a continuous function λ on Y; moreover, $\lambda(y) = \langle [X], f, y \rangle (u)$, where $\langle [X], f, y \rangle$ is a current defined by integration on $f^{-1}(y)$, with suitable multiplicities.

The motivation for this theorem was the study of algebraic or analytic cycles (in this paper called holomorphic cycles to avoid confusion with real analytic sets) by means of De Rham cohomology and currents in [11], where it was necessary to study intersection and fibering. Similar results have been obtained by others: the proof which inspired the one in this paper is that of H. Federer [8], who treats complete projective varieties. If X is a manifold and Y is normal, the result may be found in W. Stoll [23], which includes a

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