FOURIER INTEGRAL OPERATORS. I

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Preface

Pseudo-differential operators have been developed as a tool for the study of elliptic differential equations. Suitably extended versions are also applicable to hypoelliptic equations, but their value is rather limited in genuinely non-elliptic problems. In this paper we shall therefore discuss some more general classes of operators which are adapted to such applications. For these operators we shall develop a calculus which is almost as smooth as that of pseudo-differential operators. It also seems that one gains some more insight into the theory of pseudo-differential operators by considering them from the point of view of the wider classes of operators to be discussed here so we shall take the opportunity to include a short exposition.

Pseudo-differential operators as well as our Fourier integral operators are intended to make it possible to handle differential operators with variable coefficients roughly as one would handle differential operators with constant coefficients using the Fourier transformation. For example, the inhomogeneous Laplace equation

$$\Delta u = f \in C_0^\infty(\mathbf{R}^n)$$

is for $n > 2$ solved by

$$u(x) = -(2\pi)^{-n} \int e^{i\langle x,\xi\rangle} |\xi|^{-2} \hat{f}(\xi)\, d\xi,$$

where

$$\hat{f}(\xi) = \int e^{-i\langle x,\xi\rangle} f(x)\, dx$$

is the Fourier transform of f. To be able to solve arbitrary elliptic equations with variable coefficients one is led to consider more general operators of the form

$$Af(x) = (2\pi)^{-n} \int e^{i\langle x,\xi\rangle} a(x,\xi) \hat{f}(\xi)\, d\xi, \tag{0.1}$$