EICHLER INTEGRALS WITH SINGULARITIES

BY

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Introduction

Let Γ be a non-elementary Kleinian group with region of discontinuity Ω and let $q \geqslant 2$ be an integer. We shall show that there exist Eichler integrals of degree q (this term, as all others used here, will be defined below) with preassigned singularities at finitely many non-equivalent points of Ω and with preassigned parabolic singularities at finitely many non-equivalent cusps, and that these integrals have certain pleasing properties. Our results are a modest improvement of those of Ahlfors [3], who constructed Eichler integrals with preassigned poles at preassigned ordinary points in Ω. The method, however, may be of interest since it clarifies the connection between Eichler integrals with poles and generalized Beltrami coefficients (as defined in Bers [5]). That such a connection must exist becomes obvious, at least for a finitely generated group Γ, by comparing recent results of Ahlfors [3] with those of Kra [6].

1. Preliminaries

We are given a Kleinian group Γ, that is, a group of Möbius transformations $\gamma(z) = (az+b)/(cz+d)$ which acts discontinuously on some open set of the Riemann sphere $\hat{\mathbf{C}} = \mathbf{C} \cup \{\infty\}$. The largest open set Ω for which this is true is called the *region of discontinuity* of Γ; the complement $\Lambda = \hat{\mathbf{C}} - \Omega$ is nowhere dense and is called the *limit set* of Γ. We assume that Γ is *non-elementary*, that is, that Λ is infinite. The Poincaré metric $\lambda(z)|dz|$ in Ω is defined by the condition: for every component Δ of Ω, and for every universal holomorphic covering h: $U \to \Delta$ of Δ by the upper half-plane U, one has $\lambda(h(\zeta))|h'(\zeta)| = 2/|\zeta - \bar{\zeta}|$ for $h(\zeta) \in \Delta$. It is known that $\lambda(\gamma(z))|\gamma'(z)| = \lambda(z)$ for $\gamma \in \Gamma$.

The stabilizer in Γ of a point $z_0 \in \Omega$ is either the identity (then z_0 is called an *ordinary point*), or a finite cyclic group (then z_0 is called an *elliptic vertex*).