THE SUBSET OF PIECEWISE-LINEAR MAPPINGS IS DENSE IN THE SPACE OF *K*-QUASICONFORMAL MAPPINGS OF THE PLANE

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1. Introduction

For each index n from the set $\mathbf{N}$ of natural numbers, let $\mathcal{N}_n$ denote the *regular net of equilateral triangles* in the complex plane $\mathbf{C}$, whose vertice set consists of the points $[p+(\frac{1}{2}+i\sqrt{3}/2)q]2^{-n}$ with integers p and q.

A mapping φ: $\mathbf{C}\to\mathbf{C}$ is called *linear*, if there are constants a, b, $c\in\mathbf{C}$ such that $\varphi(z)=az+bz^*+c$; the superscript star denotes complex conjugation. A mapping φ: $\mathbf{C}\to\mathbf{C}$ is said to be *piecewise-linear* with respect to the net $\mathcal{N}_n$, if its restrictions to the triangles of $\mathcal{N}_n$ are linear mappings. We define the *piecewise-linearized mapping* $\varphi^{\langle n\rangle}$: $\mathbf{C}\to\mathbf{C}$ for a mapping φ: $\mathbf{C}\to\mathbf{C}$ with respect to the net $\mathcal{N}_n$ as follows: $\varphi^{\langle n\rangle}$ is piecewise-linear with respect to $\mathcal{N}_n$, and it coincides with φ on the vertice set of $\mathcal{N}_n$.

The set of continuous mappings φ: $\mathbf{C}\to\mathbf{C}$ will be considered as a *topological space with the compact-open topology*; this induces *convergence* in the sense of uniform convergence on compact subsets. *Approximation* means convergence to a given mapping. Each continuous mapping φ: $\mathbf{C}\to\mathbf{C}$ is approximated by its piecewise-linearized mappings $\varphi^{\langle n\rangle}$.

In the subspace of quasiconformal mappings of the plane, there is the problem: can each φ be approximated by φ_n which are piecewise-linear with respect to $\mathcal{N}_n$?

METHOD OF BEURLING AND AHLFORS. *Let a quasiconformal mapping φ: $\mathbf{C}\to\mathbf{C}$ have maximal dilatation* $\mathrm{K}(\varphi)<\sqrt{3}$. *Then, φ is approximated by the piecewise-linearized mappings $\varphi^{\langle n\rangle}$; $\varphi^{\langle n\rangle}$ is quasiconformal* (*Ahlfors* [2], 768; [3], 298); *$\varphi^{\langle n\rangle}$ has maximal dilatation* $\mathrm{K}(\varphi^{\langle n\rangle})\leqslant\xi[\mathrm{K}(\varphi)]$, *where ξ is a certain function involving elliptic integrals* (*Agard* [1], 739); *for each index n, there are some φ such that* $\mathrm{K}(\varphi^{\langle n\rangle})=\xi[\mathrm{K}(\varphi)]$ *holds* (*Agard* [1], 739); *moreover, there are some φ such that* $\mathrm{K}(\varphi^{\langle n\rangle})=\xi[\mathrm{K}(\varphi)]$ *holds for all indices n* ([4], 49).