RECURSIVELY ENUMERABLE DEGREES AND THE CONJUGACY PROBLEM

BY

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The principal result obtained is the theorem that for every recursively enumerable degree of unsolvability, there exists a finitely presented group whose conjugacy problem has that degree. (Parts I, II, III and IV.) In Part V this result is generalised to the theorem that certain complexes of recursively enumerable degrees of unsolvability may be obtained as the degrees of a complex of problems concerning conjugacy in a finitely presented group.

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Introduction

In 1911, Max Dehn formulated three fundamental decision problems[2] concerning groups: the word problem, the conjugacy (or transformation) problem and the isomorphism problem. These may be roughly stated as:[3] (i) Word problem for the group G—does there exist an effective method to determine of an arbitrary element W of G whether or not $W=1$ in G. (ii) Conjugacy problem for the group G—does there exist an effective method to determine of two arbitrary elements U and V of G whether or not U is conjugate to V in G. (iii) Isomorphism problem for the class C of groups—does there exist an effective method to determine of two arbitrary members G_1 and G_2 of C whether or not G_1 is isomorphic to G_2. Dehn's principal goal was the formulation of algorithms to provide effective

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(1) The material in this paper is taken from the author's Ph. D. thesis submitted to Princeton University.

(2) A decision problem is a problem of the following type. Let C be a class of entities and P a property such that every n-tuple (where n is fixed) of elements of C either does or does not enjoy P. Does there exist an effective procedure to determine of an arbitrary n-tuple $(a_1, a_2, \ldots, a_n)$ whether or not $(a_1, a_2, \ldots, a_n)$ enjoys P?

(3) A more careful statement would specify presentation of a group rather than group.