SOLUTION IN BANACH ALGEBRAS OF DIFFERENTIAL EQUATIONS WITH IRREGULAR SINGULAR POINT

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1. Introduction

We have to do with linear first-order differential equations $W'(z) = F(z)\,W(z)$, where z is a complex variable, and F and W are functions taking values in an arbitrary non-commutative Banach algebra $\mathfrak{A}$ with identity E. In [4], E. Hille has discussed the existence and nature of analytic solutions when F is holomorphic, near a regular point of F, and near a regular singular point, and has indicated how the theory will go when the equation has an irregular singular point at infinity of rank 1. The methods are adapted from the classical theory in which $\mathfrak{A}$ is the complex field $\mathfrak{C}$.

The present paper adds to the discussion with an investigation, for the cases $p \geqslant 1$, of the equation

$$z\frac{d}{dz}W(z) = (z^p P_0 + z^{p-1} P_1 + \ldots + z P_{p-1} + P_p)\,W(z), \tag{1.1}$$

a general form of first-order differential equation having an irregular singular point of rank p at infinity. Here $P_0, P_1, \ldots, P_p$ are given elements of $\mathfrak{A}$, and an analytic and algebraically regular solution W is sought which takes its values $W(z)$ in $\mathfrak{A}$. The analogous equation in which $W(z)$ is a column matrix and the P's are square matrices, over $\mathfrak{C}$, was discussed in detail by G. D. Birkhoff in [1]. He assumed P_0 to be a matrix with distinct characteristic roots, and found solutions by writing $W(z)$ as a sum of Laplace integrals in the manner of Poincaré, using these to obtain asymptotic expansions for the solutions, valid for z tending to infinity in appropriate sectors of the plane, determined by the characteristic roots of P_0. The same procedure is adopted here, under an analogous though lighter restriction on P_0: we find a solution $W(z)$ valid when z lies in appropriate sectors, corresponding to a pole $\varkappa$ of $R(\lambda, P_0)$, the resolvent of P_0, whose residue idempotent has the property of being