# THE FUNDAMENTAL GROUP OF A SURFACE, AND A THEOREM OF SCHREIER

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## Introduction

Schreier proved in [8] that a finitely generated normal subgroup $U \neq \{1\}$ of a free group $F$ is of finite index. This result was extended by Karrass and Solitar in [3], to the case when $U$ is not necessarily normal, but contains a non-trivial normal subgroup of $F$. In Topology, the free groups occur as fundamental groups of surfaces with boundary, and we here extend the result still further (Theorem 6.1) to the case when $F$ is the (non-abelian) fundamental group of any connected surface, with or without boundary, except for a Klein bottle. We use topological methods, and also the elements of Morse theory, although the latter could be eliminated. A sketch of this theory is included, however, partly for its intuitive appeal, and partly because the Morse theory picks out "stable" generators of the fundamental group, and therefore is helpful as a tool. Indeed, the author was able to use it quickly to prove Schreier's Theorem, and Theorem 3.3 below (that an open surface has free fundamental group), before knowing that proofs already existed in the literature. Our exposition is always from the point of view that it is the surface, rather than the group, which is the ultimate object of study; and we have perhaps laboured points that might irk the pure group-theorist.

## 2. Riemann surfaces

A Riemann surface $(S, \Phi)$ ([1], ch. II) is a 2-dimensional manifold $S$, for each point $x$ of which there is a "co-ordinate chart" $(U, u) \in \Phi$ (where $U$ is an open neighbourhood of $x$ in $S$, and $u$ is a homeomorphism of $U$ onto an open subset of a plane), such that if $U, V$ are overlapping charts, the homeomorphism