POLYNOMIALLY AND RATIONALLY CONVEX SETS

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On a Euclidean space of even dimension we can introduce, by a choice of complex-valued coordinate functions, $z_1, \ldots, z_n$, the structure of complex n-space, C^n. We can then associate with each compact subset, X, of our space, its *polynomial convex hull in* C^n, denoted hull(X). By definition, hull(X) *is the set of all* p *in* C^n *which satisfy the relation*

$$|f(p)| \leqslant \max_{x \in X} |f(x)|,$$

for every polynomial, $f(z_1, \ldots, z_n)$. When $X = \text{hull}(X)$, we say that X *is polynomially convex in* C^n.

Our primary object of study here is the polynomial convex hull of X. However, we have found it very helpful to consider also, as an intermediary set, R-hull(X), *the rational convex hull of* X *in* C^n. By definition, R-hull(X) *consists of all* p *in* C^n *such that*

$$|g(p)| \leqslant \max_{x \in X} |g(x)|,$$

for every rational function, g, *which is analytic about* X. For our purposes, we often prefer the alternate description of R-hull(X), (1.1), as *the set of all* p *in* C^n *for which* $f(p) \in f(X)$, *for every polynomial*, f. If $X = R\text{-hull}(X)$, we say that X *is rationally convex in* C^n. Notice that

$$X \subset R\text{-hull}(X) \subset \text{hull}(X).$$

These hulls are compact, and both inclusions can be proper.

Our aim is to understand what these hulls look like. In what sense does X "surround" them in C^n? Consider first C^1, where the complete picture is well known. There, every compact X is rationally convex (obvious), and hull(X) is formed by adjoining to X all the bounded components of its complement (classical, see (1.3)). Thus, in C^1, rational convexity

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