INVARIANTS AND FUNDAMENTAL FUNCTIONS

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Introduction

Let E be a finite-dimensional vector space over $\mathbf{R}$ and G a group of linear transformations of E leaving invariant a nondegenerate quadratic form B. The action of G on E extends to an action of G on the ring of polynomials on E. The fixed points, the *G-invariants*, form a subring. The *G-harmonic* polynomials are the common solutions of the differential equations formed by the G-invariants. Under some general assumptions on G it is shown in § 1 that the ring of all polynomials on E is spanned by products ih where i is a G-invariant and h is G-harmonic, and that the G-harmonic polynomials are of two types:

1. Those which vanish identically on the algebraic variety N_G determined by the G-invariants;

2. The powers of the linear forms given by points in N_G.

The analogous situation for the exterior algebra is examined in § 2.

Section 3 is devoted to a study of the functions on the real quadric $B=1$ whose translates under the orthogonal group $\mathbf{O}(B)$ span a finite-dimensional space. The main result of the paper (Theorem 3.2) states that (if $\dim E > 2$) these functions can always be extended to polynomials on E and in fact to $\mathbf{O}(B)$-harmonic polynomials on E due to the results of § 1.

The results of this paper along with some others have been announced in a short note [9].

§ 1. Decomposition of the symmetric algebra

Let E be a finite-dimensional vector space over a field K, let E^* denote the dual of E and $S(E^*)$ the algebra of K-valued polynomial functions on E. The sym-

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