ADDITIVE SET FUNCTIONS IN EUCLIDEAN SPACE. II

BY

C. A. ROGERS and S. J. TAYLOR

London

1. Introduction

In our previous paper [11], we discussed various decompositions of additive set functions in Euclidean space. Our main object was to show how a system of Hausdorff measures could be used to analyse a given set function, as far as is possible, into components, which were uniform in a certain sense. In the present work, we use the results of a series of papers [5, 8, 9, 12, 13] to correct and extend some of the results obtained in [11].

We continue to restrict our attention to the system $\mathfrak{F}$ of those continuous completely additive set functions F, having a finite value $F(E)$ for every set E in the field $\mathfrak{B}$ of all Borel subsets of a fixed closed rectangle I_0 in k-space. It is clear that the analysis extends immediately to σ-finite set functions, defined for Borel sets in Euclidean k-space.

In the first three sections of [11], we worked with a single Hausdorff measure function $h(t)$, and we obtained a unique decomposition of a set function F of $\mathfrak{F}$ into three components, one strongly continuous with respect to h-measure, one, not only absolutely continuous with respect to h-measure, but also concentrated on a set of σ-finite h-measure, and one concentrated on a set of zero h-measure. The extensions and refinements of this work, which we made in [13] will be vital for the sequel.

In the last three sections of [11] we introduced a system $\mathfrak{L}$ of Hausdorff measure functions $f(t)$, which was totally ordered by the relation $\prec$, defined by:

$$f \prec g, \text{ if } g(t)/f(t) \to 0, \quad \text{as } t \to +0.$$

We first studied the special case, when $\mathfrak{L}$ is the system of functions

$$t^\alpha \quad (0 < \alpha \leqslant k),$$