QUASI-INVARIANCE AND ANALYTICITY OF MEASURES ON COMPACT GROUPS

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1. Introduction

In this paper we study an extension to compact abelian groups of the two celebrated theorems of F. and M. Riesz [15] concerning analytic measures on the circle group. The content of these theorems is as follows:

Let μ be a Borel measure on the circle satisfying

$$\int_{-\pi}^{+\pi} e^{in\theta} d\mu(\theta) = 0 \quad (n = 1, 2, 3, \ldots).$$

Then

A. *μ is absolutely continuous with respect to Lebesgue measure.*

B. *If μ vanishes identically*(1) *on a set of positive Lebesgue measure, then μ must be the zero measure.*

It is not hard to see that A and B together are equivalent to the following:

The collection of Borel sets on which μ vanishes identically is invariant under rotation.

This is the assertion concerning analytic measures that we extend to compact abelian groups. Before stating this extension we make the basic definitions.

In all that follows G is a compact abelian group and $\hat{G}$ its discrete dual. An "ordering" of $\hat{G}$ is given by a fixed non-trivial(2) homomorphism ψ of $\hat{G}$ into the group R of real numbers. The mapping ψ: $\hat{G} \to R$ is a continuous homomorphism and thus induces a continuous

(1) *μ vanishes identically* on E if $\mu(F) = 0$ for all Borel subsets F of E.

(2) We assume that $\psi(\hat{G}) \neq \{0\}$.