THE SPACE OF $\mathfrak{p}$-ADIC NORMS

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Introduction

The symmetric space associated to the orthogonal group of a real indefinite quadratic form φ can be described, as is well known, as the set M of positive definite quadratic forms ψ which are minimal with respect to the property $|\varphi| \leqslant \psi$. The orthogonal group $O(\varphi)$ of the form φ acts transitively on M, and the isotropy group of any $\psi \in M$ is a maximal compact subgroup of $O(\varphi)$. A similar statement holds for the symplectic group.

A. Weil raised the question of the $\mathfrak{p}$-adic analogue of this phenomenon, and suggested the use of norms (sec. 1) in place of the positive definite quadratic forms. If φ is a non-degenerate quadratic form on a vector space E over a $\mathfrak{p}$-adic field we associate to φ the set $\mathcal{M}(\varphi)$ (see sec. 4) of norms α on E which are minimal with respect to the property $|\varphi| \leqslant \alpha^2$. Then again the orthogonal group $O(\varphi)$ of φ acts transitively on $\mathcal{M}(\varphi)$. However, the isotropy group of an element $\alpha \in \mathcal{M}(\varphi)$, while still compact, is no longer a maximal compact subgroup.

The study of norms on $\mathfrak{p}$-adic vector spaces it not new. (See for example Cohen [1] and Monna [3].) These authors were concerned with the metric topology induced on the vector space by a norm on that space. We are here concerned with the intrinsic structure that is carried by the set $\mathcal{N}(E)$ of *all* norms on a given vector space E. We define a natural metric on $\mathcal{N}(E)$, and in sec. 2 of the present paper describe some of the properties of $\mathcal{N}(E)$ as a metric space. For example, $\mathcal{N}(E)$ is a complete, locally compact arc-wise connected space, and is even contractible to a point.

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