TEICHMÜLLER SPACES OF GROUPS OF THE SECOND KIND

BY

CLIFFORD J. EARLE

Harvard University

and

Institute for Advanced Study (1)

I. Introduction

1. Let U be the upper half plane. A normalized Fuchsian group G is a discontinuous group of conformal self-mappings of U with limit points at 0, 1, and ∞. All Fuchsian groups in this paper are normalized. G is of the first (second) kind if its limit set is dense (nowhere dense) on the real axis.

Let f be a normalized quasiconformal self-mapping of U. (Throughout this paper, a normalized mapping is one that leaves 0, 1, and ∞ fixed.) f is compatible with the group G if $f\circ A\circ f^{-1}$ is conformal for all A in G. The set of mappings compatible with G is denoted by $\Sigma(G)$.

Each f in $\Sigma(G)$ induces an isomorphism of G onto $f\circ G\circ f^{-1}$. The mappings f and g induce the same isomorphism if $f\circ A\circ f^{-1}=g\circ A\circ g^{-1}$ for all A in G. This is an equivalence relation on $\Sigma(G)$. The set of equivalence classes is denoted by $S(G)$.

It is easy to see that f and g are equivalent if and only if $f=g$ on the limit set of G. Hence, for groups of the first kind, $S(G)$ equals the space $T(G)$ defined in III. If G is of the second kind, however, $T(G)$ and $S(G)$ are unequal. Thus, $T(G)$ and $S(G)$ are different generalizations of the notion of Teichmüller space to groups of the second kind. Following the terminology of Bers in [4], we shall call $T(G)$ the Teichmüller space of G. Our purpose here is to study the space $S(G)$.

Bers [4] has recently proved that $T(G)$ always carries a complex analytic structure. By contrast, if G is of the second kind, the natural structure on $S(G)$ is real analytic. Indeed, the region of discontinuity D of G is symmetric about the real axis. If one represents

---

(1) This research was supported in part by the National Science Foundation and the Air Force Office of Scientific Research.