PARAMETER ESTIMATION FOR STOCHASTIC PROCESSES

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1. Introduction

A stochastic process $[x(t), t \in I]$, or x for short, has associated with it a probability measure P_x defined on suitable subsets of the space of sample functions on I. The problems of determining when measures P_x and P_y associated with processes x and y are mutually absolutely continuous and of computing the Radon-Nikodym derivative dP_x/dP_y have been much investigated in recent years. In particular, a necessary and sufficient criterion has been given in case x and y are Gaussian for determining the mutual absolute continuity of P_x and P_y [3]. If we take I to be an interval and x and y to have zero means and correlation functions $R_x(s,t)$ and $R_y(s,t)$ whose associated integral operators on $L_2(dt, I)$ are compact, then the criterion is that $R_x^{-\frac{1}{2}} R_y R_x^{-\frac{1}{2}} - I$ have an extension to a Hilbert–Schmidt operator and under these circumstances dP_x/dP_y can be expressed in terms of the eigenfunctions and eigenvalues of this operator. In parameter estimation, however, where whole families (P_α) of measures must be considered, results of this type (which tend to involve separate calculations for each pair α_1 and α_2) often involve prohibitive amounts of calculation and also obscure the role played by the parameter itself.

In [8] we attacked this problem under the assumption that the processes x_α were gotten from each other by the application of a one-parameter group T_α of transformations acting on the sample functions of the process. Specifically, we assumed given an algebra F of bounded random variables on which T_α operated as a group of automorphisms (intuitively $(T_\alpha f)(x) = f(T_\alpha x)$) such that the derivative $DT_\alpha f(x) = \partial T_\alpha f(x)/\partial \alpha$ existed and was uniformly bounded in α and x. It was shown there that the existence of a random variable φ satisfying $\int \varphi f dP_x = \int Df dP_x$ for all f in F implied the existence of a strongly continuous one-parameter group $[V(\alpha) \mid \alpha \geqslant 0]$ of contractions on $L_1(P_x)$

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