Schauder's existence theorem for α-Dini continuous data

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1.

In 1934 J. Schauder (see [14]) proved his well known theorem on the existence of classical solutions to linear elliptic partial differential equations of second order. In this article we shall establish the following improvement of Schauder's theorem: A classical solution exists, if the given data (coefficients, boundary values, right hand side) are uniformly continuous and their modulus of continuity is bounded by some function θ which owns the following two properties:

i) $$\int_0^1 \theta(\tau)/\tau \, d\tau < \infty.$$

ii) There is an $0<\alpha<1$, such that $\theta(\tau)/\tau^\alpha$ is *monotonically decreasing* on some interval $(0, T]$.

For notations we refer to **16.** below. We shall always use the summation convention.

2.

In order to give a precise statement, let us introduce the following notions: Let $\zeta\colon [0, \infty)\to[0, \infty)$ be a monotonically increasing function, $\lim_{t\to 0+} \zeta(t)=0$, $\zeta(0)=0$, $\zeta(t)>0$, if $t>0$; B a real Banach space equiped with the norm $\|\cdot\|_B$; $A\subset\mathbf{R}^n$ a nonvoid, open set. $C^{0,\zeta}(A)=C^{0,\zeta}(A, B)$ is the set of all bounded continuous functions $f\colon A\to B$, for which

$$(2.1)\qquad [f]_\zeta := \sup_{x,y\in A, x\neq y} \|f(x)-f(y)\|_B/\zeta(\|x-y\|)$$

is finite. It is easy to prove, that $C^{0,\zeta}(A)$ becomes a Banach space under the norm:

$$(2.2)\qquad \|f\|_{0,\zeta} := \|f\|_0+[f]_\zeta.$$

If $k\in\mathbf{N}$, let $C^{k,\zeta}(A)$ be the set of k-times uniformly continuously differentiable