Commutator and other second order estimates in real interpolation theory

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1. Introduction and summary

The main theme of this paper is that an operator T which is bounded on a parametrized family of Banach spaces will exhibit further properties (in addition to boundedness) on individual spaces in the family. In [RW] this theme was developed for families of spaces obtained by complex interpolation. The results there concerned a mapping δ which is the differential of the natural map between the spaces of the complex interpolation family. Although δ itself is generally unbounded and nonlinear, boundedness results for the commutator between δ and various linear maps T were obtained. In this paper similar studies are made for spaces obtained by real interpolation. Although there are strong analogies between the two cases, the details are very different. Hence it was rather a surprise to us that in some (but not all) cases the specific results obtained here are the same as those in [RW]. Our results here involve maps Ω derived from an analysis of the real interpolation process. As was true with δ, Ω is generally unbounded and non-linear. In some cases Ω is the same as the map δ obtained in the complex interpolation theory.

In Section 2 we recall the basics of real interpolation theory. We introduce the K and J functionals and the associated interpolation spaces. We also introduce other functionals E and F which are closely related to K and J and which lead to alternative forms of Ω.

In Section 3 we give our abstract results. The basic philosophy is that interpolation spaces are constructed by finding efficient ways of decomposing functions into pieces which can then be effectively studied separately. If there is also a bounded linear operator T in the picture, the decomposition can be done before or after

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