

## Erratum

# Erratum to “Compact Operators for Almost Conservative and Strongly Conservative Matrices”

S. A. Mohiuddine,<sup>1</sup> M. Mursaleen,<sup>2</sup> and A. Alotaibi<sup>1</sup>

<sup>1</sup> Department of Mathematics, Faculty of Science, King Abdulaziz University, P.O. Box 80203, Jeddah 21589, Saudi Arabia

<sup>2</sup> Department of Mathematics, Aligarh Muslim University, Aligarh 202002, India

Correspondence should be addressed to M. Mursaleen; mursaleenm@gmail.com

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We redefine the space  $f$  and state the results of [1] in this light.

Let  $\mathcal{B}$  be a semigroup of positive regular matrices  $B = (b_{nk})$ .

A bounded sequence  $x = (x_k)$  is said to be  $\mathcal{B}$ -almost convergent to the value  $l$  if and only if  $t_{pn}(x) \rightarrow l$ , as  $p \rightarrow \infty$  uniformly in  $n$ , where

$$t_{pn}(x) = \frac{1}{p+1} \sum_{m=0}^p B_{m+n}(x); \quad (p, n \in \mathbb{N}), \quad (1)$$

and  $B_n(x) = \sum_{k=1}^{\infty} b_{nk} x_k$  which is  $B$ -transform of a sequence  $x$  (see Mursaleen [2]). The number  $l$  is called the generalized limit of  $x$ , and we write  $l = f - \lim x$ . We write

$$f = \left\{ x \in \ell_{\infty} : \lim_{p \rightarrow \infty} t_{pn}(x) = L \text{ uniformly in } n \right\}. \quad (2)$$

Using the idea of  $\mathcal{B}$ -almost convergence, we define the following.

An infinite matrix  $A = (a_{nk})_{n,k=1}^{\infty}$  is said to be  $\mathcal{B}$ -almost conservative if  $Ax \in f$  for all  $x \in c$ , and we denote it by  $A \in (c, f)$ . An infinite matrix  $A = (a_{nk})_{n,k=1}^{\infty}$  is said to be  $\mathcal{B}$ -strongly conservative if  $Ax \in c$  for all  $x \in f$ , and we denote it by  $A \in (f, c)$ .

Now, we restate Theorem 11 and Theorem 15 of [1] as follows, respectively.

**Theorem 11.** Let  $A = (a_{nk})$  be a  $\mathcal{B}$ -almost conservative matrix. Then, one has

$$0 \leq \|L_A\|_{\chi} \leq \limsup_{n \rightarrow \infty} \left( \sum_{k=1}^{\infty} |\tilde{a}_{nk}| \right), \quad (3)$$

$$L_A \text{ is compact if } \lim_{n \rightarrow \infty} \left( \sum_{k=1}^{\infty} |\tilde{a}_{nk}| \right) = 0,$$

where  $\tilde{a}_{nk} = \sum_{j=1}^{\infty} a_{nj} b_{jk}$ .

*Proof.* It follows on the same lines as of Theorem 11 [1] by only replacing  $a_{nk}$  by  $\tilde{a}_{nk}$ .  $\square$

**Theorem 15.** Let  $B$  be a normal positive regular matrix. Let  $A = (a_{nk})$  be an infinite matrix. Then, one has the following.

(i) If  $A \in (f, c_0)$ , then

$$\|L_A\|_{\chi} = \limsup_{n \rightarrow \infty} \left( \sum_{k=1}^{\infty} |\tilde{a}_{nk}| \right). \quad (4)$$

(ii) If  $A \in (f, c)$ , then

$$\begin{aligned} & \frac{1}{2} \cdot \limsup_{n \rightarrow \infty} \left( \sum_{k=1}^{\infty} |\tilde{a}_{nk} - \alpha_k| \right) \\ & \leq \|L_A\|_{\chi} \leq \limsup_{n \rightarrow \infty} \left( \sum_{k=1}^{\infty} |\tilde{a}_{nk} - \alpha_k| \right), \end{aligned} \quad (5)$$

where  $\alpha_k = \lim_{n \rightarrow \infty} \tilde{a}_{nk}$  for all  $k \in \mathbb{N}$ .

(iii) If  $A \in (f, \ell_\infty)$ , then

$$0 \leq \|L_A\|_\chi \leq \limsup_{n \rightarrow \infty} \left( \sum_{k=1}^{\infty} |\hat{a}_{nk}| \right), \quad (6)$$

where  $\hat{A} = (\hat{a}_{nk})$  is the composition of the matrices  $A$  and  $B^{-1}$ ; that is,  $\hat{a}_{nk} = \sum_{j=1}^{\infty} a_{nj} b_{jk}^{-1}$ .

*Proof.* It follows on the same lines as Theorem 15 of [1] by only replacing  $a_{nk}$  by  $\hat{a}_{nk}$ .  $\square$

*Remark 1* (see [2]). If  $\mathcal{B}$  consists of the iterates of the operator  $T$  defined on  $\ell_\infty$  by  $Tx = (x_{\sigma(n)})$ , where  $\sigma$  is an injection of the set of positive integers into itself having no finite orbits, then  $\mathcal{B}$ -invariant mean is reduced to the  $\sigma$ -mean and  $\mathcal{B}$ -almost convergence is reduced to  $\sigma$ -convergence. In this case, our results are reduced to the results of [3].

## References

- [1] S. A. Mohiuddine, M. Mursaleen, and A. Alotaibi, "Compact operators for almost conservative and strongly conservative matrices," *Abstract and Applied Analysis*, vol. 2014, Article ID 567317, 6 pages, 2014.
- [2] M. Mursaleen, "On  $\mathcal{A}$ -invariant mean and  $\mathcal{A}$ -almost convergence," *Analysis Mathematica*, vol. 37, no. 3, pp. 173–180, 2011.
- [3] M. Mursaleen and A. K. Noman, "On  $\sigma$ -conservative matrices and compact operators on the space  $V_\sigma$ ," *Applied Mathematics Letters: An International Journal of Rapid Publication*, vol. 24, no. 9, pp. 1554–1560, 2011.