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Research Article

Global Finite-Time Output Feedback Stabilization for a Class of Uncertain Nonholonomic Systems

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This paper investigates the problem of global finite-time stabilization by output feedback for a class of nonholonomic systems in chained form with uncertainties. By using backstepping recursive technique and the homogeneous domination approach, a constructive design procedure for output feedback control is given. Together with a novel switching control strategy, the designed controller renders that the states of closed-loop system are regulated to zero in a finite time. A simulation example is provided to illustrate the effectiveness of the proposed approach.

1. Introduction

Over the past decade, nonholonomic systems have attracted much attention because they can be used to model many real systems, such as mobile robots, car-like vehicle, and underactuated satellites. An important feature of a nonholonomic system is that the number of its inputs is less than the number of its degree of freedom, which makes the control problems of a nonholonomic system challenging. As pointed out by Brockett in [1], there does not exist a pure-state feedback control law for a nonholonomic system such that its state converges to its equilibrium. To overcome this difficulty, with the effort of many researchers a number of intelligent approaches have been proposed, which can be classified into discontinuous control laws [2, 3], time-varying control laws [4–6], and hybrid control laws [7, 8]; see the survey paper [9] for more details and references therein. Considering the difficulty of measuring full states and the inevitability of uncertainties in engineering practice, the output feedback issue of nonholonomic systems with drift uncertainties has recently been studied [10–16]. However, it should be mentioned that the aforementioned works only consider the feedback stabilizer that makes the trajectories of the systems converge to the equilibrium as the time goes to infinity.

Compared to the asymptotic stabilization, the finite-time stabilization, which renders the trajectories of the closed-loop systems convergent to the origin in a finite time, has many advantages such as fast response, high tracking precision, and disturbance-rejection properties [17]. Hence, it is more meaningful to investigate the finite-time stabilization problem than the classical asymptotical stability. In recent years, the problem of finite-time stabilization for nonlinear systems has been studied and some interesting results have been obtained [18–25]. However, the finite-time stabilization of nonholonomic systems is a relatively new problem. In fact, even in the case of finite-time stabilization using state feedback, there are very few results in the literature [26–28]. In the case when parts of the states are not measurable, to stabilize a nonholonomic system in a finite time only using limited measurable states becomes challenging.

To illustrate the difficulties in finite-time control of nonholonomic systems via output feedback, let us consider a problem of finite time stabilizing the following simple system at the origin:

$$\dot{x}_0 = u_0, \qquad \dot{x}_1 = x_1 u_0, \qquad \dot{x}_2 = u_1, \tag{1}$$

where x_0 and x_1 are measurable and x_2 is not available for feedback.

In discontinuous approach, as seen, for example, in [26–28], assuming that $x_0(t_0) \neq 0$ one might design the control u_0 as follows:

$$u_0(x_0) = -k_0 x_0^{\alpha_0}, \quad 0 < \alpha_0 = \frac{\alpha_1}{\alpha_2} < 1, \tag{2}$$

where k_0 is a positive design parameter and α_i, $i = 1, 2$, are positive odd numbers. It is easy to verify that the u_0 in (2) renders x_0 globally converging to zero in a finite time T_0.

Next, we need to stabilize the x-subsystem

$$\dot{x}_1 = -k_0 x_0^{\alpha_0} x_2, \qquad \dot{x}_2 = u_1 \tag{3}$$

within a settling time T_1 satisfying $T_1 < T_0$. By introducing the input-state-scaling transformation $x_1 = x_1/u_0$ and $z_2 = x_2$, the system (3) can be rewritten as

$$\dot{z}_1 = z_2 + \frac{\dot{u}_0}{u_0} z_1, \qquad \dot{z}_2 = u_1. \tag{4}$$

However, the system (3) possesses the time-varying coefficient $-k_0 x_0^{\alpha_0}$ (or the system (4) dissatisfies the low-order growth condition), which renders the existing finite-time control methods highly difficult to the control problem of the x-subsystem or even inapplicable. To the best of the authors' knowledge, there is no result referred to the finite-time stabilization of nonholonomic systems by output feedback.

Motivated by the aforementioned discussion, in this paper we aim to tackle this challenging question and provide a solution to the problem of global finite-time output feedback stabilization for nonholonomic systems with uncertainties by applying the homogeneous domination approach. The main contribution of this paper is twofold. (i) Compared to the existing output feedback stabilization results for nonholonomic systems, the finite-time stabilizer proposed in this paper leads to faster convergence rate. (ii) As the common assumption to guarantee the existence of global finite-time output feedback stabilizer for a nonlinear system, the low-order growth (the order less than one) of system nonlinearities renders the discontinuous change of coordinates (i.e., the σ-process) inapplicable to the finite-time output feedback control problem of the nonholonomic systems, even the ideal chained systems, and how to deal with this constitutes one of the main contributions of this paper.

The rest of this paper is organized as follows. Section 2 provides the problem formation and preliminary knowledge. Section 3 presents the control design procedure and the main result, while Section 4 gives a simulation example to illustrate the theoretical finding of this paper. Finally, concluding remarks are proposed in Section 5.

2. Problem Formulation and Preliminaries

In this paper, we consider the following uncertain nonholonomic systems:

$$\begin{aligned} \dot{x}_0 &= d_0 u_0 + \phi_0(t, x_0), \\ \dot{x}_i &= d_i x_{i+1} u_0 + \phi_i(t, x_0, x), \quad i = 1, \dots, n-1, \\ \dot{x}_n &= d_n u_1 + \phi_n(t, x_0, x), \\ y &= (x_0, x_1)^T, \end{aligned} \tag{5}$$

where $(x_0, x)^T = (x_0, x_1, \dots, x_n)^T \in R^{n+1}$, $u = (u_0, u_1)^T \in R^2$, $y \in R^2$ are the system state, control input, and system output, respectively, d_i's are disturbed virtual control coefficients, and ϕ_i's denote the input and states driven uncertainties, which are called the nonlinear drifts of the system (5).

The objective of this paper is to design an output feedback controller in the form

$$\dot{\hat{x}} = \vartheta(\hat{x}, y), \qquad u_0 = u_0(x_0), \qquad u_1 = u_1(\hat{x}, y), \tag{6}$$

such that the finite-time regulation of the states is achieved; that is, $\lim_{t\to T}(|x_0(t)| + |x(t)|) = 0$ and $(x_0(t), x(t)) = (0, 0)$ for any $t \ge T$, where T is a finite settling time.

To this end, the following assumptions regarding system (5) are imposed.

Assumption 1. For $i = 0, 1, \dots, n$, there are positive constants c_{i1} and c_{i2} such that

$$c_{i1} \le d_i \le c_{i2}. \tag{7}$$

Assumption 2. For ϕ_0, there is a positive constant a such that

$$|\phi_0(t, x_0)| \le a|x_0|. \tag{8}$$

Assumption 3. For $i = 1, \dots, n$, there are constants $b > 0$ and $\tau \in (-1/n, 0)$ such that

$$|\phi_i(t, x_0, x)| \le b\left(|x_1|^{(r_i+\tau)/r_1} + \cdots + |x_i|^{(r_i+\tau)/r_i}\right), \tag{9}$$

where $r_i = 1 + (i-1)\tau$.

For simplicity, in this paper we assume $\tau = -p/q$ with p being any even integer and q being any odd integer. Based on this, we know that $r_i \in (0, 1)$ is a ratio of two positive odd integers.

Remark 4. Assumptions 1-2 are common and similar to the one usually imposed on the nonlinear systems [10]. Relatively speaking, Assumption 3 seems to be quite restrictive; however, it plays an essential role in ensuring the existence of finite-time output feedback stabilizer for nonholonomic system (5). Furthermore, it is worth pointing out that there are a number of nonlinear functions such as $\sin x$ and $\ln(1 + x^2)$ that can be bounded by a function $|x|^m$ for any constant $m \in (0, 1)$ actually satisfying this assumption.

The following definitions and lemmas will serve as the basis of the coming control design and performance analysis.

Definition 5 (see [17]). Consider a system

$$\dot{x} = f(x) \quad \text{with } f(0) = 0, \; x \in R^n, \tag{10}$$

where $f : U_0 \to R^n$ is continuous with respect to x on an open neighborhood U_0 of the origin $x = 0$. The equilibrium $x = 0$ of the system is (locally) finite-time stable if it is Lyapunov stable and finite-time convergent in a neighborhood $U \in U_0$ of the origin. By "finite-time convergence," we mean that if, for any initial condition, $x(0) \in U$, there is a settling time $T > 0$, such that every solution $x(t)$ with $x(0)$ as its initial condition of (10) is well defined with $x(0) \in U \setminus \{0\}$ for $t \in [0, T)$ and satisfies $\lim_{t\to T} x(t) = 0$ and $x(t) = 0$ for any $t \ge T$. If $U = U_0 = R^n$, the origin is a globally finite-time stable equilibrium.

Lemma 6 (see [17]). *Consider the nonlinear system described in* (10). *Suppose that there is a C^1 function $V(x)$ defined in a neighborhood $\widehat{U} \in R^n$ of the origin, real numbers $c > 0$, and $0 < \alpha < 1$, such that*

(i) *$V(x)$ is positive definite on $\widehat{U}$;*

(ii) *$\dot{V}(x) + cV^{\alpha}(x) \leq 0$, $\forall x \in \widehat{U}$.*

Then, the origin of system (10) *is locally finite-time stable with*

$$T \leq \frac{V^{1-\alpha}(x(0))}{c(1-\alpha)} \tag{11}$$

for initial condition $x(0)$ in some open neighborhood $U \in \widehat{U}$ of the origin. If $U = R^n$ and $V(x)$ is also radially unbounded (i.e., $V(x) \to +\infty$ as $x \to +\infty$), the origin of system (10) *is globally finite-time stable.*

Definition 7 (see [29]). Weighted homogeneity: for fixed coordinates $(x_1, \ldots, x_n) \in R^n$ and real numbers $r_i > 0$, $i = 1, \ldots, n$, one has the following.

(i) The dilation $\Delta_{\varepsilon}(x)$ is defined by $\Delta_{\varepsilon}(x) = (\varepsilon^{r_1}x_1, \ldots, \varepsilon^{r_n}x_n)$ for any $\varepsilon > 0$, where r_i is called the weights of the coordinates. For simplicity, we define dilation weight $\Delta = (r_1, \ldots, r_n)$.

(ii) A function $V \in (R^n, R)$ is said to be homogeneous of degree τ if there is a real number $\tau \in R$ such that $V(\Delta_{\varepsilon}(x)) = \varepsilon^{\tau}V(x_1, \ldots, x_n)$ for any $x \in R^n \backslash \{0\}$, $\varepsilon > 0$.

(iii) A vector field $f \in (R^n, R^n)$ is said to be homogeneous of degree τ if there is a real number $\tau \in R$ such that $f_i(\Delta_{\varepsilon}(x)) = \varepsilon^{\tau + r_i} f_i(x)$, for any $x \in R^n \setminus \{0\}$, $\varepsilon > 0$, $i = 1, \ldots, n$.

(iv) A homogeneous p-norm is defined as $\|x\|_{\Delta,p} = (\sum_{i=1}^{n} |x_i|^{p/r_i})^{1/p}$ for all $x \in R^n$, for a constant $p \geq 1$. For simplicity, in this paper, one chooses $p = 2$ and writes $\|x\|_{\Delta}$ for $\|x\|_{\Delta,2}$.

Lemma 8 (see [30]). *Suppose that $V : R^n \to R$ is a homogeneous function of degree τ with respect to the dilation weight Δ. Then the following hold.*

(i) *$\partial V/\partial x_i$ is homogeneous of degree $\tau - r_i$ with r_i being the homogeneous weight of x_i.*

(ii) *There is a constant c such that $V(x) \leq c\|x\|_{\Delta}^{\tau}$. Moreover, if $V(x)$ is positive definite, then $\underline{c}\|x\|_{\Delta}^{\tau} \leq V(x)$, where $\underline{c}$ is a constant.*

Lemma 9 (see [31]). *For $x \in R$, $y \in R$, and $p \geq 1$ which is a constant, the following inequalities hold:*

$$\begin{gathered} |x + y|^p \leq 2^{p-1}|x^p + y^p|, \\ (|x| + |y|)^{1/p} \leq |x|^{1/p} + |y|^{1/p} \leq 2^{(p-1)/p}(|x| + |y|)^{1/p}. \end{gathered} \tag{12}$$

If $p \geq 1$ is odd, then

$$\begin{gathered} |x - y|^p \leq 2^{p-1}|x^p - y^p|, \\ \left|x^{1/p} - y^{1/p}\right| \leq 2^{(p-1)/p}(|x - y|)^{1/p}. \end{gathered} \tag{13}$$

Lemma 10 (see [32]). *Let x, y be real variables; then for any positive real numbers a, m, and n, one has*

$$\begin{aligned} &a|x|^m|y|^n \\ &\quad \leq b|x|^{m+n} + \frac{n}{m+n}\left(\frac{m+n}{m}\right)^{-m/n} a^{(m+n)/n} b^{-m/n} |y|^{m+n}, \end{aligned} \tag{14}$$

where $b > 0$ is any real number.

Lemma 11 (see [33]). *For $x, y \in R$ and positive real number p, the following inequality holds:*

$$\begin{aligned} \left|x^p - y^p\right| &\leq p\,|x - y|\left|x^{p-1} + y^{p-1}\right| \\ &\leq c\,|x - y|\left|(x - y)^{p-1} + y^{p-1}\right|, \end{aligned} \tag{15}$$

where $c = p$ for $1 < p \leq 2$ and $c = p2^{p-1}$ for $p > 2$.

3. Finite-Time Output Feedback Controller Design

In this section, we give a constructive procedure for the finite-time stabilizer of system (5) by output feedback. The design of finite-time output feedback controller is divided into the following two steps.

(i) We first stabilize the x-subsystem in a finite time by output feedback.

(ii) Then we design a controller such that the x_0-subsystem is finite-time stable.

3.1. Finite-Time Output Feedback Stabilization of the x-Subsystem. For the x_0-subsystem, we choose the control u_0 as

$$u_0 \equiv u_0^*, \tag{16}$$

where u_0^* is a positive constant. In this case, the x_0-subsystem becomes

$$\dot{x}_0 = d_0 u_0^* + \phi_0(t, x_0). \tag{17}$$

Noting that $\phi_0(t, x_0)$ satisfies the linear growth condition, it is easy to obtain that the solution of x_0-subsystem is bounded, for any given finite time $t_s > 0$. Hence, x_0 is well defined on $[0, t_s]$. Under the control law (16), the x-subsystem can be written as

$$\begin{aligned} \dot{x}_i &= d_i u_0^* x_{i+1} + \phi_i(t, x_0, x), \quad i = 1, \ldots, n-1, \\ \dot{x}_n &= d_n u_1 + \phi_n(t, x_0, x). \end{aligned} \tag{18}$$

Next we consider the finite-time output feedback stabilizer for system (18). For convenience, we define the following change of coordinates:

$$\zeta_1 = x_1, \qquad \zeta_i = d_1 \cdots d_{i-1} u_0^{*i-1} x_i, \quad i = 2, \ldots, n \tag{19}$$

under which system (18) is transformed into

$$\dot{\zeta}_i = \zeta_{i+1} + f_i(t, x_0, \zeta), \quad i = 1, \dots, n-1,$$
$$\dot{\zeta}_n = du_1 + f_n(t, x_0, \zeta), \tag{20}$$

where $d = d_1 \cdots d_n u_0^{*n-1}$, $f_i = d_1 \cdots d_{i-1} u_0^{*i-1} \phi_i$, and the state $\zeta_1 = x_1$ is measurable.

Remark 12. It is worth pointing out that, in terms of the transformation (19), the stabilizing control design of system (18) is equivalent to that of system (20). Thus, in what follows, we turn to designing the output feedback stabilizing controller for system (20) rather than (18). Moreover, with the help of Assumptions 1 and 3, it can be verified that f_i, $i = 1, \dots, n$, satisfy

$$|f_i(t, x_0, \zeta)| \le b\left(|\zeta_1|^{(r_i+\tau)/r_1} + \cdots + |\zeta_i|^{(r_i+\tau)/r_i}\right) \tag{21}$$

with a new growth rate b.

To construct a global output feedback controller for system (20), we will employ the homogeneous domination approach introduced in [34]. We will first construct specifically a homogeneous output feedback controller for the nominal system without considering perturbing terms f_i's. Then, we utilize a scaling gain in the controller to dominate the uncertain nonlinearities f_i's.

3.1.1. Homogeneous Output Feedback Control of the Nominal System. In this subsection, we will construct an output feedback stabilizer for the following nominal system:

$$\dot{z}_i = z_{i+1}, \quad i = 1, \dots, n-1, \quad \dot{z}_n = dv. \tag{22}$$

The design of output feedback controller is divided into two steps. In Step A, we suppose that all the states are measurable and develop a recursive design method to explicitly construct a state feedback control law for system (22). Then in Step B, by constructing a nonsmooth reduced-order observer, we design an output feedback controller.

(A) State Feedback Controller Design

Step 1. Choose the Lyapunov function $V_1 = x_1^2/2$. Clearly, the first virtual controller

$$z_2^* = -nz_1^{r_2} := -\beta_1 \xi_1^{r_2} \tag{23}$$

with $\xi_1 = z_1$ and $\beta_1 = n$ renders

$$\dot{V}_1 \le -n\xi_1^{2+\tau} + \xi_1 (z_2 - z_2^*). \tag{24}$$

Step i ($2 \le i \le n$). In this step, we can obtain the following property.

Proposition 13. *For the ith Lyapunov function defined by*

$$V_i = V_{i-1} + \int_{z_i^*}^{z_i} \left(s^{1/r_i} - z_i^{*1/r_i}\right)^{2-r_i} ds \tag{25}$$

under the coordinate transformation

$$z_k^* = -\beta_{k-1}\xi_{k-1}^{r_k}, \qquad \xi_k = z_k^{1/r_k} - z_k^{*1/r_k}, \quad k = 2, \dots, i \tag{26}$$

there exists the C^0 virtual controller $z_{i+1}^* = -\beta_i \xi_i^{1/r_{i+1}}$ such that

$$\dot{V}_i \le -(n-i+1)\left(\xi_1^{2+\tau} + \cdots + \xi_i^{2+\tau}\right) + \xi_i^{2-r_i}\left(z_{i+1} - z_{i+1}^*\right), \tag{27}$$

where $\beta_j > 0$, $j = 1, \dots, i$ are constants.

Proof. The detailed proof can be found in [20] and hence is omitted here. □

From the inductive steps, we can design

$$\begin{aligned} z_{n+1}^* &= -\beta_n \xi_n^{r_n+\tau} \\ &= -\beta_n \Big(z_n^{1/r_n} + \beta_{n-1}^{1/r_n} \\ &\qquad \times \left(z_{n-1}^{1/r_{n-1}} + \cdots + \beta_2^{1/r_3}\left(z_2^{1/r_2} + \beta_1^{1/r_2} z_1\right)\right)\Big)^{r_n+\tau} \\ &= -\beta_n \left(\overline{\beta}_n z_n^{1/r_n} + \overline{\beta}_{n-1} z_{n-1}^{1/r_{n-1}} + \cdots + \overline{\beta}_1 z_1\right)^{r_n+\tau}, \end{aligned} \tag{28}$$

where

$$\overline{\beta}_i = \begin{cases} \beta_{n-1}^{1/r_n} \cdots \beta_i^{1/r_{i+1}}, & i = 1, \dots, n-1 \\ 1, & i = n \end{cases} \tag{29}$$

such that

$$\dot{V}_n \le -\left(\xi_1^{2+\tau} + \cdots + \xi_n^{2+\tau}\right) + d\xi_n^{2-r_n}\left(v - z_{n+1}^*\right). \tag{30}$$

(B) Output Feedback Controller Design. Since $z_2, \dots, z_n$ are unmeasurable, we construct a homogeneous observer

$$\dot{\eta}_i = -l_{i-1}\hat{z}_i, \qquad \hat{z}_i = \left(\eta_i + l_{i-1}\hat{z}_{i-1}\right)^{r_i/r_{i-1}}, \quad i = 2, \dots, n, \tag{31}$$

where $\hat{z}_1 = z_1$ and $l_i > 0$; $i = 1, \dots, n-1$ are the gains to be determined. By the certainty equivalence principle, we can replace z_i with $\hat{z}_i$ in (28) and obtain an output feedback controller

$$v(\hat{z}) = -\beta_n \left(\overline{\beta}_n \hat{z}_n^{1/r_n} + \overline{\beta}_{n-1}\hat{z}_{n-1}^{1/r_{n-1}} + \cdots + \overline{\beta}_1 z_1\right)^{r_n+\tau}, \tag{32}$$

where $\hat{z} = (z_1, \hat{z}_2, \dots, \hat{z}_n)$.

Considering

$$W_i = \int_{\gamma_i^{(2-r_{i-1})/r_{i-1}}}^{z_i^{(2-r_{i-1})/r_i}} \left(s^{r_{i-1}/(2-r_{i-1})} - \gamma_i\right) ds, \tag{33}$$

where $\gamma_i = \eta_i + l_{i-1}z_{i-1}$, and setting $e_i = (z_i - \hat{z}_i)^{1/r_i}$, for $i = 2, \dots, n$, from (22), (31), and (33), it follows that

$$\begin{aligned} \dot{W}_i &= \frac{2-r_{i-1}}{r_i} z_i^{(2-r_{i-1}-r_i)/r_i}\left(z_i^{r_{i-1}/r_i} - \gamma_i\right) z_{i+1} \\ &\quad - l_{i-1} e_i^{r_i}\left(z_i^{(2-r_{i-1})/r_i} - \hat{z}_i^{(2-r_{i-1})/r_i}\right) \\ &\quad - l_{i-1} e_i^{r_i}\left(\hat{z}_i^{(2-r_{i-1})/r_i} - \gamma_i^{(2-r_{i-1})/r_{i-1}}\right), \end{aligned} \tag{34}$$

where $z_{n+1} = v(\hat{z})$.

Each term on the right-hand side of (34) can be estimated by the following propositions whose proofs are given in the Appendix.

Proposition 14. *There exists a positive constant λ_i such that*

$$-l_{i-1}e_i^{r_i}\left(z_i^{(2-r_{i-1})/r_i}-\hat{z}_i^{(2-r_{i-1})/r_i}\right)\leq -l_{i-1}\lambda_i e_i^{2+\tau}. \tag{35}$$

Proposition 15. *For $i=2,\ldots,n-1$,*

$$\begin{aligned}&\frac{2-r_{i-1}}{r_i}z_i^{(2-r_{i-1}-r_i)/r_i}\left(z_i^{r_{i-1}/r_i}-\gamma_i\right)z_{i+1}\\&\quad\leq\frac{1}{12}\sum_{j=i-1}^{i+1}\xi_j^{2+\tau}+m_ie_i^{2+\tau}+g_i\left(l_{i-1}\right)e_{i-1}^{2+\tau},\end{aligned} \tag{36}$$

where g_i is a continuous function of l_{i-1}, $m_i>0$ is a constant, and $g_2=0$.

Proposition 16. *For the controller $v(\hat{z})$, one obtains*

$$\begin{aligned}&\frac{2-r_{n-1}}{r_n}z_n^{(2-r_{n-1}-r_n)/r_n}\left(z_n^{r_{n-1}/r_n}-\gamma_n\right)v\left(\hat{z}\right)\\&\quad\leq\frac{1}{8}\sum_{j=1}^{n}\xi_j^{2+\tau}+c\sum_{i=2}^{n}e_i^{2+\tau}+g_n\left(l_{n-1}\right)e_{n-1}^{2+\tau},\end{aligned} \tag{37}$$

where g_n is a continuous function of l_{n-1} and $c>0$ is a constant.

Proposition 17. *For $i=3,\ldots,n$,*

$$\begin{aligned}&-l_{i-1}e_i^{r_i}\left(\hat{z}_i^{(2-r_{i-1})/r_i}-\gamma_i^{(2-r_{i-1})/r_{i-1}}\right)\\&\quad\leq\frac{1}{16}\left(\xi_{i-1}^{2+\tau}+\xi_i^{2+\tau}\right)+e_i^{2+\tau}+h_i\left(l_{i-1}\right)e_{i-1}^{2+\tau},\end{aligned} \tag{38}$$

where h_i is a continuous function of l_{i-1}.

Choosing $W=\sum_{i=2}^{n}W_i$, by Propositions 14–17, we get

$$\begin{aligned}\dot{W}=&\frac{1}{2}\sum_{i=1}^{n}\xi_i^{2+\tau}+\left(-l_1\lambda_2+m_2+c+g_3\left(l_2\right)+h_3\left(l_2\right)\right)e_2^{2+\tau}\\&+\sum_{i=3}^{n-1}\left(-l_{i-1}\lambda_i+m_i+1+c+g_{i+1}\left(l_i\right)+h_{i+1}\left(l_i\right)\right)e_i^{2+\tau}\\&+\left(-l_{n-1}\lambda_n+1+c\right)e_n^{2+\tau}.\end{aligned} \tag{39}$$

By (28), (32), and Assumption 1, we can estimate $d\xi_i^{2-r_n}(v-z_{n+1}^*)$ in (29) by the following proposition, whose proof is given in the Appendix.

Proposition 18. *There exists a positive constant μ such that*

$$d\xi_i^{2-r_n}\left(v-z_{n+1}^*\right)\leq\frac{1}{4}\sum_{i=1}^{n}\xi_i^{2+\tau}+\mu\sum_{i=2}^{n}e_i^{2+\tau}, \tag{40}$$

where g_n is a continuous function of l_{n-1}.

With the help of Proposition 18, defining $U=V_n+W$, combining (29) and (39), and recursively choosing

$$\begin{aligned}&l_{n-1}\geq\lambda_n^{-1}\left(\frac{1}{4}+1+c+\mu\right),\\&l_{i-1}\geq\lambda_i^{-1}\left(\frac{1}{4}+m_i+1+c+\mu+g_{i+1}\left(l_i\right)+h_{i+1}\left(l_i\right)\right),\\&\qquad\qquad i=n-1,\ldots,3,\\&l_1\geq\lambda_2^{-1}\left(\frac{1}{4}+m_2+c+\mu+g_3\left(l_2\right)+h_3\left(l_2\right)\right)\end{aligned} \tag{41}$$

we obtain

$$\dot{U}=-\frac{1}{4}\sum_{i=1}^{n}\xi_i^{2+\tau}-\frac{1}{4}\sum_{i=2}^{n}e_i^{2+\tau}. \tag{42}$$

Since U is positive definite and proper with respect to $Z=(z_1,\ldots,z_n,\eta_2\ldots,\eta_n)^T$, (42) implies that the closed-loop system can be rewritten as the following compact form:

$$\dot{Z}=F\left(Z\right)=\left(z_2,\ldots,z_n,dv,\dot{\eta}_2\ldots,\dot{\eta}_n\right)^T \tag{43}$$

which is homogeneous with the dilation weight

$$\begin{aligned}\Delta&=\left(\underbrace{r_1,\ldots,r_n}_{\text{for } z_1,\ldots,z_n},\underbrace{r_1,\ldots,r_{n-1}}_{\text{for } \eta_2\ldots,\eta_n}\right)\\&=\left(\underbrace{1,\ldots,1+(n-1)\tau}_{\text{for } z_1,\ldots,z_n},\underbrace{1,\ldots,1+(n-2)\tau}_{\text{for } \eta_2\ldots,\eta_n}\right).\end{aligned} \tag{44}$$

It can be shown that (43) is homogeneous of degree τ. In addition, U is homogeneous of degree 2. By Lemma 8, there is a constant $\bar{c}_1$, such that

$$U\leq\bar{c}_1\|Z\|_\Delta^2, \tag{45}$$

where $\bar{c}_1>0$ and $\|Z\|_\Delta=\sqrt{(\sum_{i=1}^{2n-1}|Z_i|^{2/r_i})}$. Similarly, since the right-hand side of (42) is homogeneous of degree $2+\tau$, by Lemma 8 there is a constant $\bar{c}_2$ such that

$$\frac{\partial U}{\partial Z}F\left(Z\right)\leq-\frac{1}{4}\sum_{i=1}^{n}\xi_i^{2+\tau}-\frac{1}{4}\sum_{i=2}^{n}e_i^{2+\tau}\leq-\bar{c}_2\|Z\|_\Delta^{2+\tau}. \tag{46}$$

Combining (45) and (46), it can be deduced from (42) that

$$\dot{U}\leq-kU^{(2+\tau)/2} \tag{47}$$

for a constant $k>0$. By Lemma 6 with $\alpha=(2+\tau)/2<1$, the closed-loop system is globally finite-time stable.

Remark 19. It should be pointed out that the output feedback controller (32) is only continuous (rather than continuously differentiable) due to the presence of the powers $r_n+\tau$, which is less than one. As a consequence, the closed-loop system (22) and (32) is not locally Lipschitz. Therefore, the uniqueness of the solution of system (22) and (32) is not guaranteed. Fortunately, as shown in the work [35], the existence of the solution can still be guaranteed for a continuous system without Lipschitz condition.

3.1.2. Homogeneous Output Feedback Control of the System (20). Together with the homogeneous controller and observer established previously, in this subsection we are ready to use the homogeneous domination approach to globally stabilize (20) via output feedback under (21). First, we introduce the change of coordinates

$$z_i = \frac{\eta_i}{L^{i-1}}, \quad i = 2, \dots, n, \qquad v = \frac{u_1}{L^n}, \tag{48}$$

where $L \geq 1$ is a constant to be determined later. Under (48), system (20) can be rewritten as

$$\dot{z}_i = L z_{i+1} + \frac{f_i(\cdot)}{L^{i-1}}, \quad i = 1, \dots, n-1, \qquad \dot{z}_n = L d v + + \frac{f_n(\cdot)}{L^{n-1}}. \tag{49}$$

Now we construct an observer with a gain L as follows:

$$\dot{\eta}_i = -L l_i \hat{z}_i, \qquad \hat{z}_i = \left(\eta_i + l_{i-1}\hat{z}_{i-1}\right)^{r_i/r_{i-1}}, \quad i = 2, \dots, n. \tag{50}$$

In addition, we design u_1 using the same construction of (32), specifically,

$$u_1 = -L^n \beta_n \left(\overline{\beta}_n \hat{z}_n^{1/r_n} + \overline{\beta}_{n-1}\hat{z}_{n-1}^{1/r_{n-1}} + \cdots + \overline{\beta}_1 z_1\right)^{r_n+\tau}. \tag{51}$$

Now, the closed-loop system (49)–(51) can be written as

$$\dot{Z} = LF(Z) + \left(f_1(\cdot), \frac{f_2(\cdot)}{L}, \dots, \frac{f_n(\cdot)}{L^{n-1}}, 0, \dots, 0\right)^T. \tag{52}$$

Hence, it can be concluded from (46) that

$$\dot{U} \leq -L\overline{c}_2 \|Z\|_\Delta^{2+\tau} + \frac{\partial U}{\partial Z}\left(f_1(\cdot), \frac{f_2(\cdot)}{L}, \dots, \frac{f_n(\cdot)}{L^{n-1}}, 0, \dots, 0\right)^T. \tag{53}$$

From (21), (48), and $L \geq 1$, we can find constants $\delta_i > 0$ and $\alpha_i < 1$ such that

$$\left|\frac{f_i(\cdot)}{L^{i-1}}\right| \leq b\sum_{j=1}^{i} \frac{L^{(j-1)(r_i+\tau)/r_j}}{L^{i-1}} \left|z_j\right|^{(r_i+\tau)/r_j} \leq \delta_i L^{\alpha_i}\|Z\|_\Delta^{r_i+\tau}. \tag{54}$$

Noting that, for $i = 1, \dots, n$, $\partial U/\partial Z_i$ is homogeneous of degree $2 - r_i$, we know that

$$\frac{\partial U}{\partial Z_i}\left(\left|z_1\right|^{(r_i+\tau)/r_1} + \cdots + \left|z_i\right|^{(r_i+\tau)/r_i}\right) \tag{55}$$

is homogeneous of degree $2 + \tau$.

With (54) and (55) in mind, we can find a positive constant c_i such that

$$\left|\frac{\partial U}{\partial Z_i}\right|\left|\frac{f_i(\cdot)}{L^{i-1}}\right| \leq c_i L^{\alpha_i}\|Z\|_\Delta^{2+\tau}. \tag{56}$$

Substituting (56) into (53) yields

$$\begin{aligned} \dot{U} &\leq -L\left(\overline{c}_2 - \sum_{i=1}^{n} c_i L^{\alpha_i - 1}\right)\|Z\|_\Delta^{2+\tau} \\ &\leq -L\left(\overline{c}_2 - \sum_{i=1}^{n} c_i L^{\alpha_{\max} - 1}\right)\|Z\|_\Delta^{2+\tau}, \end{aligned} \tag{57}$$

where $\alpha_{\max} = \max_{1\leq i\leq n}\{\alpha_i\} < 1$. Apparently, by choosing a large enough L, the right-hand side of (57) is negative definite.

Furthermore, it can be deduced from (57) that there is a constant $\overline{c}_3$ such that

$$\dot{U} \leq -\overline{c}_3 U^{(2+\tau)/2}. \tag{58}$$

By Lemma 6 ($U = V$, $c = \overline{c}_3$, and $\alpha = (2+\tau)/2 < 1$), (58) leads to the conclusion that the closed-loop system (20), (50), and (51) is globally finite-time stable, which yields that system (18) can be globally finite-time stabilized by the output feedback. In addition, the settling time T_1 satisfies

$$T_1 \leq \frac{-2U^{(-\tau)/2}(0)}{\overline{c}_3 \tau}. \tag{59}$$

3.2. Finite-Time Output Feedback Stabilization of the x_0-Subsystem. From Section 3.1, we know that $x(t) \equiv 0$ when $t \geq T_1$. Therefore, we just need to stabilize the x_0-subsystem in a finite time. When $t \geq T_1$, for the x_0-subsystem, we can take the following control law:

$$u_0(x_0) = g_0(x_0)\, x_0^{\alpha_0}, \quad 0 < \alpha_0 = \frac{\alpha_1}{\alpha_2} < 1, \qquad g_0(x_0) = -\frac{1}{c_{01}}\left(k_0 + \phi_0(x_0)\right), \tag{60}$$

where k_0 is a positive design constant, α_i, $i = 1, 2$ are positive odd numbers, and $\phi_0(x_0) \geq a|x_0|^{1-\alpha_0} \geq 0$ is a smooth function. For instance, we can simply choose $\phi_0(x_0) = a(1 + x_0^2)$.

Taking the Lyapunov function $V_0 = x_0^2/2$, a simple computation gives

$$\dot{V}_0 \leq -k_0 x_0^{1+\alpha_0} \leq -k_0 V_0^{(1+\alpha_0)/2}. \tag{61}$$

Thus, by Lemma 6, x_0 tends to 0 within a settling time denoted by T_2 and

$$T_2 \leq \frac{2V_0^{(1-\alpha_0)/2}(0)}{k_0(1-\alpha_0)}. \tag{62}$$

Up to now, we have finished the finite-time output feedback stabilizing controller design of the system (5). Consequently, the following theorem can be obtained to summarize the main result of the paper.

Theorem 20. *Under Assumptions 1–3, if the proposed control design procedure together with the above switching control strategy is applied to system* (5)*, then, for any initial conditions in the state space $(x_0, x) \in R^{n+1}$, the closed-loop system is globally finite-time regulated at origin.*

4. Simulation Example

To verify our proposed controller, we consider the following low-dimensional system:

$$\begin{aligned}
\dot{x}_0 &= d_0 u_0 + \theta_0(t)\, x_0, \\
\dot{x}_1 &= d_1 x_2 u_0 + \theta_1(t) \ln\left(1 + x_1^2\right), \\
\dot{x}_2 &= d_2 u_1 + \theta_2 \sin x_1,
\end{aligned} \tag{63}$$

where d_i, $i = 0, 1, 2$ are unknown constants and $\theta_i(t)$, $i = 0, 1, 2$ are unknown functions.

It should be mentioned that, when $d_1 = d_2 = 1$ and $\theta_0(t) = \theta_1(t) = \theta_2(t) = 0$, the system (63) collapses into a third-order chained form system which can be viewed as the bilinear model of a mobile robot with small angle measurement error (see [10, 36] for more details). This means that the system (63) is a simple one; however, it comes from real world.

For simplicity, it is assumed that $d_i \in [0.5, 1]$ and $|\theta_i(t)| \leq 1$, $i = 1, 2, 3$. From this and Remark 4, it is not difficult to verify that Assumptions 1–3 hold. Firstly, we define the control law $u_0 = 1$ and introduce the change of coordinates

$$\zeta_1 = x_1, \qquad \zeta_2 = d_1 x_2 \tag{64}$$

under which the x-subsystem of (63) is transformed into

$$\begin{aligned}
\dot{\zeta}_1 &= \zeta_2 + \theta_1(t) \ln\left(1 + \zeta_1^2\right), \\
\dot{\zeta}_2 &= d_1 d_2 u_1 + d_1 \theta_2(t) \sin \zeta_1.
\end{aligned} \tag{65}$$

If we pick $\tau = -2/5$, the dilation is defined as $r_1 = 1$ and $r_2 = 3/5$. Then, according to the design procedure shown in Section 3, we can explicitly construct an output feedback controller for system (65). We can choose specifically

$$\begin{aligned}
\dot{\eta}_2 &= -L l_1 \hat{z}_2, \qquad \hat{z}_2 = \left(\eta_2 + l_1 z_1\right)^{3/5}, \\
u_1 &= -L^2 \beta_2 \left(\hat{z}_2^{5/3} + \beta_1^{5/3} z_1\right)^{1/5}
\end{aligned} \tag{66}$$

with appropriate positive constants l_1, β_1, β_2, and a large enough gain L such that output feedback controller (66) renders the system (65) (i.e., the x-subsystem of (63)) globally finite-time stable with a settling time T_1.

Then, when $t \geq T_1$, for the x_0-subsystem, we switch the control input u_0 to

$$u_0(x_0) = -2 k_0 x_0^{1/3}, \tag{67}$$

where k_0 is a positive design constant.

In the simulation, we assume $d_0 = d_1 = d_2 = 1$ and $\theta_0(t) = \theta_1(t) = \theta_2(t) = \sin t$. When $(x_0(0), x_1(0), x_2(0), \eta_2(0)) = (0, 1, -1, 0)$, by choosing the gains for the output laws as $L = 2$, $\beta_1 = 2$, $\beta_2 = 9$, $l_1 = 20$, and $k_0 = 1$, the simulation shown in Figure 1 demonstrates the global finite-time stability property of the closed-loop system (63)–(67).

Remark 21. Although system (63) was asymptotically stabilized by the existing output feedback controller in [10, 13], the system (66)-(67) is the first output feedback controller which globally finite-time stabilizes system (63). Compared to the existing asymptotical stabilization results, the proposed controller demonstrates more advantages such as faster convergence rates, higher accuracies, and better disturbance rejection properties [17].

5. Conclusion

This paper has solved the problem of global finite-time output feedback stabilization for a class of nonholonomic systems in chained form with uncertainties. With the help of backstepping recursive technique and the homogeneous domination approach, a constructive design procedure for output feedback control is given. It is shown that the designed control laws can guarantee that the closed-loop system states are globally finite-time regulated to zero. In this direction, there are still remaining problems to be investigated. For example, an interesting research problem is how to design a finite-time output feedback stabilizing controller for nonholonomic systems in stochastic setting.

Appendix

Proof of Proposition 14. By Lemma 9, one obtains

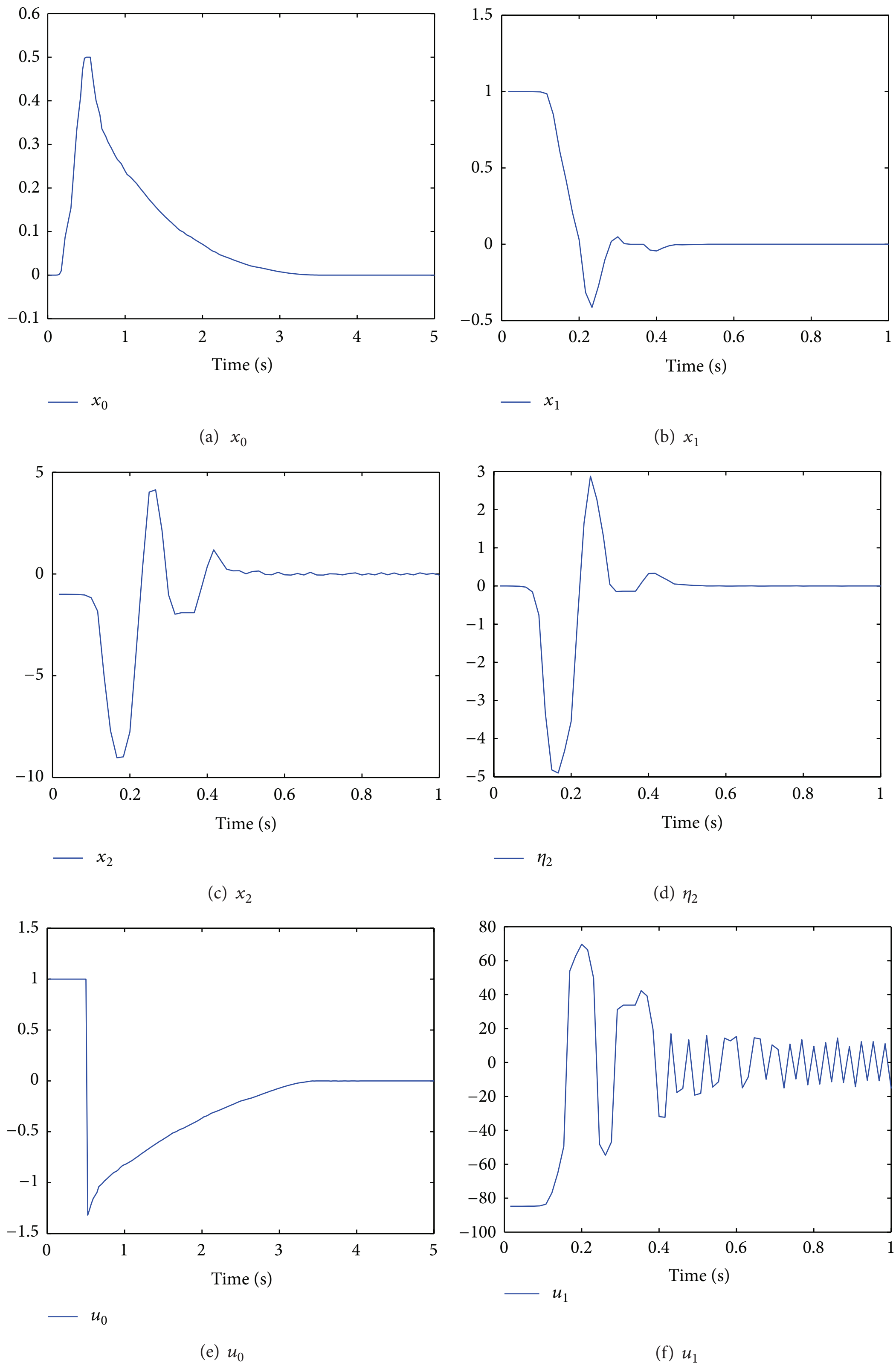
$$\begin{aligned}
&- l_{i-1} e_i^{r_i} \left(z_i^{(2-r_{i-1})/r_i} - \hat{z}_i^{(2-r_{i-1})/r_i}\right) \\
&\quad = -l_{i-1} e_i^{r_i} \left(\left(z_i^{1/r_i}\right)^{2-r_{i-1}} - \left(\hat{z}_i^{1/r_i}\right)^{2-r_{i-1}}\right) \\
&\quad \leq -l_{i-1} \lambda_i e_i^{2+\tau},
\end{aligned} \tag{A.1}$$

where $\lambda_i > 0$ is a constant. □

Proof of Proposition 15. Using $\gamma_i = \sigma_i + l_{i-1} z_{i-1}$, (26), (31), and Lemmas 9–11, it follows that

$$\begin{aligned}
&\frac{2 - r_{i-1}}{r_i} z_i^{(2-r_{i-1}-r_i)/r_i} \left(z_i^{(r_{i-1})/r_i} - \gamma_i\right) z_{i+1} \\
&\quad = \frac{2 - r_{i-1}}{r_i} \left(\xi_{i+1} - \beta_i^{1/r_{i+1}} \xi_i\right)^{r_{i+1}} \left(\xi_i - \beta_{i-1}^{1/r_i} \xi_i\right)^{2-r_{i-1}-r_i} \\
&\qquad \times \left(\left(z_i^{r_{i-1}/r_i} - \hat{z}_i^{r_{i-1}/r_i}\right) - l_{i-1}\left(z_{i-1} - \hat{z}_{i-1}\right)\right) \\
&\quad \leq k_{i3} \left(\left|\xi_{i+1}\right|^{r_{i+1}} + \left|\xi_i\right|^{r_{i+1}}\right) \\
&\qquad \times \left(\left|\xi_i\right|^{2-r_{i-1}-r_i} + \left|\xi_{i-1}\right|^{2-r_{i-1}-r_i}\right) \\
&\qquad \times \Big[\left|e_i\right|^{r_{i-1}} + \left|e_i\right|^{r_i} \left(\left|\xi_i\right|^{r_{i-1}-r_i} + \left|\xi_{i-1}\right|^{r_{i-1}-r_i}\right) \\
&\qquad\qquad + l_{i-1} \left|e_{i-1}\right|^{r_{i-1}}\Big] \\
&\quad \leq \frac{1}{12} \sum_{j=i-1}^{i+1} \xi_j^{2+\tau} + m_i e_i^{2+\tau} + g_i\left(l_{i-1}\right) e_{i-1}^{2+\tau},
\end{aligned} \tag{A.2}$$

where $k_{i3} > 0$, $m_i > 0$ are constants and g_i is a continuous function of l_{i-1}. By $e_1 = 0$, one has $g_2 = 0$. □



(a) x_0

(b) x_1

(c) x_2

(d) η_2

(e) u_0

(f) u_1

Figure 1: The responses of the closed-loop system (63)–(67).

Proof of Proposition 16. By (26), (32), $e_i = (z_i - \hat{z}_i)^{1/r_i}$, and the definition of the homogeneous norm, one gets

$$\begin{aligned} |v(\hat{z})| &\le k_{n4}\|\hat{z}\|_{\Delta}^{r_n+\tau} \\ &= k_{n4}\left(\sum_{i=1}^{n}|\hat{z}_i|^{2/r_i}\right)^{(r_n+\tau)/2} \\ &\le \bar{k}_{n4}\left(\sum_{i=1}^{n}|z_i|^{(r_n+\tau)/r_i} + \sum_{i=1}^{n}|e_i|^{r_n+\tau}\right) \\ &\le \tilde{k}_{n4}\left(\sum_{i=1}^{n}|\xi_i|^{r_n+\tau} + \sum_{i=1}^{n}|e_i|^{r_n+\tau}\right), \end{aligned} \tag{A.3}$$

where k_{n4}, $\bar{k}_{n4}$, and $\tilde{k}_{n4}$ are positive constants.

Similar to (A.2), with the use of Lemmas 9–11 and (A.3), (37) holds immediately. □

Proof of Proposition 17. From $e_i = (z_i - \hat{z}_i)^{1/r_i}$, (26), and Lemma 9, there is a positive constant k_{i5} such that

$$|\hat{z}_i| = |z_i - e_i^{r_i}| \le |z_i| + |e_i|^{r_i} \le k_{i5}\left(|\xi_{i-1}|^{r_i} + |\xi_i|^{r_i} + |e_i|^{r_i}\right). \tag{A.4}$$

According to $\gamma_i = \sigma_i + l_{i-1}z_{i-1}$, (31), (A.4), and Lemmas 9–11, one obtains

$$\begin{aligned} &-l_{i-1}e_i^{r_i}\left(\hat{z}_i^{(2-r_{i-1})/r_i} - \gamma_i^{(2-r_{i-1})/r_{i-1}}\right) \\ &\quad= l_{i-1}e_i^{r_i}\Big((\sigma_i + l_{i-1}\hat{z}_{i-1})^{(2-r_{i-1})/r_{i-1}} \\ &\qquad\qquad -(\sigma_i + l_{i-1}z_{i-1})^{(2-r_{i-1})/r_{i-1}}\Big) \\ &\quad\le k_{i5}l_{i-1}^2|e_i|^{r_i}|e_{i-1}|^{r_{i-1}} \\ &\qquad\times\Big(l_{i-1}^{(2-2r_{i-1})/r_{i-1}}|e_{i-1}|^{2-2r_{i-1}} + |\xi_{i-1}|^{2-2r_{i-1}} \\ &\qquad\qquad + |\xi_i|^{2-2r_{i-1}} + |e_i|^{2-2r_{i-1}}\Big) \\ &\quad\le \frac{1}{16}\left(\xi_{i-1}^{2+\tau} + \xi_i^{2+\tau}\right) + e_i^{2+\tau} + h_i(l_{i-1})\,e_{i-1}^{2+\tau}, \end{aligned} \tag{A.5}$$

where h_i is a continuous function of l_{i-1}. □

Proof of Proposition 18. By (26), (31), and Lemmas 9–11, it follows that

$$\begin{aligned} &d\xi_i^{2-r_n}(v - z_{n+1}^*) \\ &\quad= -d\xi_i^{2-r_n}\beta_n\Big[\left(\bar{\beta}_n z_n^{1/r_n} + \bar{\beta}_{n-1}z_{n-1}^{1/r_{n-1}} + \cdots + \bar{\beta}_1 z_1\right)^{r_n+\tau} \\ &\qquad\qquad - \Big(\bar{\beta}_n\hat{z}_n^{1/r_n} + \bar{\beta}_{n-1}\hat{z}_{n-1}^{1/r_{n-1}} \\ &\qquad\qquad\qquad + \cdots + \bar{\beta}_1 z_1\Big)^{r_n+\tau}\Big] \\ &\quad\le k_{n6}|\xi_i|^{2-r_n} \\ &\qquad\times\left[\sum_{i=2}^{n}|z_i - \hat{z}_i|\left(|z_i - \hat{z}_i|^{(1-r_i)/r_i} + |z_i|^{(1-r_i)/r_i}\right)\right]^{r_n+\tau} \\ &\quad= \bar{k}_{n6}|\xi_i|^{2-r_n} \\ &\qquad\times\left[\sum_{i=2}^{n}|e_i|^{r_i}\left(|e_i|^{1-r_i} + |\xi_{i-1}|^{1-r_i} + |\xi_i|^{1-r_i}\right)\right]^{r_n+\tau} \\ &\quad\le \frac{1}{4}\sum_{i=1}^{n}\xi_i^{2+\tau} + \mu\sum_{i=2}^{n}e_i^{2+\tau}, \end{aligned} \tag{A.6}$$

where k_{n6}, $\bar{k}_{n6}$, and μ are positive constants. □

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