

## MULTIPLIERS WHICH ARE NOT COMPLETELY BOUNDED

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ABSTRACT. For an infinite compact Abelian group  $G$  and  $1 < p < 2$ , it was shown in [9] that there exists a  $L^p(G)$  multiplier which is not completely bounded. In this note, we show that in infinite every locally compact Abelian group  $G$  there is a  $L^p(G)$  multiplier which is not completely bounded.

### 1. Introduction

Let  $G$  be a locally compact Abelian group and  $\hat{G}$  be its dual. A bounded linear operator  $T$  on  $L^p(G)$ ,  $1 \leq p < \infty$ , is called a  $L^p$ -multiplier if  $T$  commutes with translation operator  $\tau_x$  for each  $x \in G$ . We will denote the space of all  $L^p$ -multipliers by  $M_p(G)$ . It is well known that  $T \in M_p(G)$  corresponds to a symbol  $m \in L^\infty(\hat{G})$  such that  $\widehat{Tf} = m\hat{f}$  for all  $f \in L^1(G) \cap L^2(G)$ . Sometimes we prefer to work with symbol  $m$  in place of  $T$ .

We now briefly recall the natural operator space structure on  $L^p(X)$ -spaces, where  $X$  is a  $\sigma$ -finite measure space. For details see [9, Chapter 2].

A  $C^*$ -algebra has a canonical operator space structure. We consider this canonical operator space structure on  $L^\infty(X)$ . The operator space structure on  $L^1(X)$  is inherited from the dual of  $L^\infty(X)$ . By [3], with this operator space structure we have  $L^1(X)^* = L^\infty(X)$  complete isometrically. Now by [9] the couple  $(L^\infty(X), L^1(X))$  is compatible for operator space interpolation. We consider  $L^p(X) = (L^\infty(X), L^1(X))_{\frac{1}{p}}$  with the operator space structure as the interpolating operator space structure from [9].

If a  $L^p$  multiplier  $T$  is completely bounded in the above mentioned operator space structure of  $L^p$ , we call this a  $cb$ -multiplier on  $L^p(G)$ . We will denote the space of all  $cb$ -multipliers on  $L^p(G)$  by  $M_p^{cb}(G)$ . Throughout this paper, we will assume  $G$  to be infinite.

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We will be repeatedly using following important result of Pisier [9] which provides a characterization of completely bounded maps on  $L^p(G), 1 \leq p < \infty$ .

PROPOSITION 1.1. [9] *Let  $S_p$  be the space of Schatten  $p$ -class operators on  $l_2(\mathbb{Z})$ . A linear map  $T : L^p(X) \rightarrow L^p(X)$  is completely bounded if and only if the mapping  $T \otimes I_{S_p}$  is bounded on  $L^p(X, S_p)$ . Moreover,*

$$\|T\|_{cb} = \|T \otimes I_{S_p}\|_{L^p(X, S_p) \rightarrow L^p(X, S_p)}.$$

Using above result one can see that  $M_p^{cb}(G) = M_p(G)$  for  $p = 1$  or  $2$ . A natural question arises what happens for  $1 < p < 2$ . It was shown in [9, Proposition 8.1.3] that for a compact Abelian group  $G$  and for  $1 < p < 2$ , the inclusion  $M_p^{cb}(G) \subsetneq M_p(G)$  is strict. The purpose of this note is to show that this inclusion is strict for any locally compact Abelian group.

In [9, Proposition 8.1.3], an explicit construction of  $m \in M_p(\mathbb{T}) \setminus M_p^{cb}(\mathbb{T})$  is provided for circle group  $\mathbb{T}$ . We briefly describe the construction below. Let  $1 < p < 2$  and  $\Lambda = \{3^{2i} + 3^{2j+1} : i, j \in \mathbb{N}\}$ . Then  $\Lambda$  is a  $\Lambda_{p'}$  set in the sense that for any  $f \in L^2(\mathbb{T})$  whose Fourier transform is supported in  $\Lambda$ , we have  $\|f\|_{p'} \leq C_{p'} \|f\|_2$  for some constant  $C_{p'}$  depending on  $p'$ . It is well known that for  $S_p, p \neq 2$  the canonical basis  $(e_{ij})$  is not an unconditional basis. Furthermore, it is shown (see [9, Lemma 8.1.5]) that for any  $n \in \mathbb{N}$  there exist complex scalars  $\{z_{ij} : i, j = 1, 2, \dots, n\}$  and an element  $x = \sum_{i,j} x_{ij} e_{ij}$  in the unit ball of  $S_p, p \neq 2$ , such that  $\|\sum_{i,j} z_{ij} x_{ij} e_{ij}\|_{S_p} = n^{|\frac{1}{2} - \frac{1}{p}|}$ . Define  $m$  on  $\mathbb{Z}$  by

$$m(n) = \begin{cases} z_{ij} & \text{if } n = 3^{2i} + 3^{2j+1}, \\ 0 & \text{otherwise.} \end{cases}$$

Since  $\Lambda$  is a  $\Lambda_{p'}$  set, we have  $m \in M_p(\mathbb{T})$ . However, the choice of  $(z_{ij})$  and from Proposition 1.1 it follows that  $m \notin M_p^{cb}(\mathbb{T})$ . Same conclusion can be drawn for compact Abelian group with the help of  $\Lambda_p$  sets.

For non-compact locally compact Abelian group  $G$ , we will approach the problem of strict inclusion via suitable transference techniques. In order to explain this explicitly in the case of  $G = \mathbb{R}$ , we need following terminology.

DEFINITION 1.2. Let  $\phi_n(x) = \frac{1}{2n} \chi_{[-n,n]} * \chi_{[-n,n]}(x)$ . A function  $m \in L^\infty(\mathbb{R})$  is said to be normalized (with respect to  $\{\phi_n\}$ ) if  $\lim_{n \rightarrow \infty} (\hat{\phi}_n * m)(x) = m(x)$  for all  $x \in \mathbb{R}$ .

In particular, a bounded continuous function is always normalized. Our main result in this note is the following theorem, which is  $cb$  version of deLeeuw's theorem [5].

THEOREM 1.3. *Let  $m$  be normalized and  $m \in M_p^{cb}(\mathbb{R})$ . Then  $m|_{\mathbb{Z}} \in M_p^{cb}(\mathbb{T})$  with  $\|T_{m|_{\mathbb{Z}}}\|_{cb} \leq \|T_m\|_{cb}$ .*

Theorem 1.3 provides an explicit construction of  $\tilde{m} \in M_p(\mathbb{R}) \setminus M_p^{cb}(\mathbb{R})$ . Consider the multiplier  $m \in M_p(\mathbb{T}) \setminus M_p^{cb}(\mathbb{T})$  described above. We extend  $m$

to  $\mathbb{R}$  as a piece-wise linear continuous function  $\tilde{m}$ . Since  $\tilde{m}$  is bounded and continuous it is normalized with respect to  $\{\phi_n\}$  (as in Definition 1.2) and  $\tilde{m} \in M_p(\mathbb{R})$  (see [8]). If  $\tilde{m} \in M_p^{cb}(\mathbb{R})$ , then by Theorem 1.3 we have  $m \in M_p^{cb}(\mathbb{T})$ . This contradicts the construction of  $m$  and hence  $\tilde{m} \in M_p(\mathbb{R}) \setminus M_p^{cb}(\mathbb{R})$ .

In Section 2 we prove Theorem 1.3 and in Section 3 we will show strict inclusion  $M_p^{cb}(G) \subsetneq M_p(G)$  for arbitrary non-compact locally compact Abelian group.

### 2. Transference of $cb$ multipliers

Our main tool in this paper is a transference result (Theorem 2.1) for  $cb$ -multipliers. This is a  $cb$  version of transference couple result by Berkson, Paluszynki and Weiss [2]. Techniques adopted to prove our result is along the same line as in [2] with appropriate use of Proposition 1.1.

**THEOREM 2.1.** *Let  $G$  be an amenable group and  $X$  a  $\sigma$ -finite measure space. Let  $R, S : G \rightarrow \mathcal{CB}(L^p(X))$  satisfy the following conditions:*

- (i) *for each  $f \in L^p(X)$ ,  $u \mapsto R_u f$  and  $u \mapsto S_u f$  are strongly continuous maps.*
- (ii)  *$C_R = \sup_{u \in G} \|R_u\|_{cb} < \infty$  and  $C_S = \sup_{u \in G} \|S_u\|_{cb} < \infty$ .*
- (iii)  *$S_u R_v = R_{uv}$  and  $S_u S_v = S_{uv}$  for all  $u, v \in G$ .*

*Let  $k \in L^1(G)$  have compact support. Consider the operator  $H_k$  on  $L^p(X)$  defined by*

$$H_k f(\cdot) = \int_G k(u) R_{u^{-1}} f \, du(\cdot).$$

*If  $N_p(k)$  denotes the  $cb$ -norm of the convolution operator  $F \mapsto k * F$  on  $L^p(G)$ , then we have  $H_k$  is a  $cb$  map on  $L^p(X)$  and  $\|H_k\|_{cb} \leq C_S C_R N_p(k)$ .*

*Proof.* In view of Proposition 1.1 in order to show that  $H_k$  is a  $cb$  map on  $L^p(X)$ , we need to show  $H_k \otimes I$  is a bounded map on  $L^p(S_p)$  where  $I$  is the identity map on  $S_p$ .

It is easy to see that  $(S_{v^{-1}} \otimes I)(S_v \otimes I)(H_k \otimes I) = H_k \otimes I$  on  $L^p(G) \otimes S_p$ . For  $l \in \mathbb{N}$ ,  $\{f_n\}_{n=1}^l \subseteq C_c(X)$  and  $\{h_n\}_{n=1}^l \subseteq S_p$  we have

$$(S_v \otimes I)(H_k \otimes I) \left( \sum_n f_n \otimes h_n(x) \right) = \sum_n \int_G k(u) R_{vu^{-1}} f_n(\cdot) \, du \otimes h_n(x).$$

Hence,

$$\begin{aligned} & \left\| (H_k \otimes I) \left( \sum_n f_n \otimes h_n \right) \right\|_{L^p(S_p)}^p \\ & \leq C_s^p \left\| \sum_n \int_G k(u) R_{vu^{-1}} f_n(\cdot) \, du \otimes h_n \right\|_{L^p(S_p)}^p \quad (\text{by Proposition 1.1}) \\ & = C_s^p \int_X \left\| \sum_n \int_G k(u) R_{vu^{-1}} f_n(\cdot) \, du \otimes h_n(x) \right\|_{S_p}^p \, dx. \end{aligned}$$

The above inequality is true for all  $v \in G$ . For a suitable neighborhood  $V$  of identity in  $G$ , which we will choose later, we have,

$$\begin{aligned} & \left\| (H_k \otimes I) \left( \sum_n f_n \otimes h_n \right) \right\|_{L^p(S_p)}^p \\ & \leq C_s^p \frac{1}{|V|} \int_X \int_V \left\| \sum_n \int_G k(u) R_{vu^{-1}} f_n(\cdot) du \otimes h_n(x) \right\|_{S_p}^p dv dx \\ & \leq C_s^p \frac{1}{|V|} \int_X \int_G \left\| \sum_n \int_G k(u) R_{vu^{-1}} f_n(x) du h_n \right\|_{S_p}^p dv dx \\ & \leq C_s^p \frac{1}{|V|} \int_X \int_G \left\| \sum_n (k * F_n(\cdot, x) \otimes h_n)(v) \right\|_{S_p}^p dv dx, \end{aligned}$$

where  $F_n(v, x) = \chi_{VK^{-1}}(v) R_v f_n(x)$  and  $K$  is the support of  $k$ . Observe that  $F_n(\cdot, x) \in L^p(G)$  for a.e.  $x$ . Hence,

$$\begin{aligned} & \left\| (H_k \otimes I) \left( \sum_n f_n \otimes h_n \right) \right\|_{L^p(S_p)}^p \\ & \leq C_s^p \frac{1}{|V|} \int_X N_p(k)^p \int_G \left\| \sum_n (F_n(\cdot, x) \otimes h_n)(v) \right\|_{S_p}^p dv dx \\ & = C_s^p \frac{1}{|V|} N_p(k)^p \int_G \int_X \left\| \sum_n F_n(v, x) h_n \right\|_{S_p}^p dx dv \\ & = C_s^p \frac{1}{|V|} N_p(k)^p \int_G \int_X \left\| \sum_n \chi_{VK^{-1}}(v) R_v f_n(x) h_n \right\|_{S_p}^p dx dv \\ & = C_s^p \frac{1}{|V|} N_p(k)^p \int_G \chi_{VK^{-1}}(v) \int_X \left\| \sum_n R_v f_n(x) h_n \right\|_{S_p}^p dx dv \\ & = C_s^p \frac{1}{|V|} N_p(k)^p \int_G \chi_{VK^{-1}}(v) \left\| (R_v \otimes I) \sum_n f_n \otimes h_n \right\|_{L^p(S_p)}^p dv \\ & \leq C_s^p C_R^p \frac{1}{|V|} N_p(k)^p |VK^{-1}| \left\| \sum_n f_n \otimes h_n \right\|_{L^p(S_p)}^p. \end{aligned}$$

Last inequality follows from Proposition 1.1. Since  $G$  is amenable, for every  $\varepsilon > 0$  and compact set  $K$  we can choose a neighborhood  $V$  of identity such that  $\frac{|VK^{-1}|}{|V|} < 1 + \varepsilon$ . Hence,  $\|(H_k \otimes I)(\sum_n f_n \otimes h_n)\|_{L^p(S_p)} \leq C_s C_R N_p(k) \|\sum_n f_n \otimes h_n\|_{L^p(S_p)}$ .  $\square$

We will need following lemmas to prove Theorem 1.3. These results are analogue of usual  $L^p$  multipliers and can be of independent interest.

LEMMA 2.2. *Let  $N \in \mathbb{N}$ . If*

$$\begin{aligned} & \left| \int_{\hat{G}} \left( \sum_{n=1}^N \hat{f}'_n(\xi) h'_n \right) \left( \sum_{n=1}^N m(\xi) \hat{f}_n(\xi) h_n \right) d\xi \right| \\ & \leq c_m \left\| \sum_{n=1}^N f_n \otimes h_n \right\|_{L^p(S_p)} \left\| \sum_{n=1}^N f'_n \otimes h'_n \right\|_{L^{p'}(S_{p'})} \end{aligned}$$

for all  $f_n, f'_n \in C_c(G)$  and for all  $h_n \in S_p, h'_n \in S_{p'}, n = 1, 2, \dots, N$ , where  $c_m$  is a constant which does not depend on  $N$  then  $m \in M_p^{cb}(G)$ .

*Proof.* The result follows from the density of the elements  $\sum_{n=1}^N f_n \otimes h_n, N \in \mathbb{N}, f_n \in C_c(G), h_n \in S_p$  in  $L^p(S_p)$  and duality of  $L^p(S_p)$  and  $L^{p'}(S_{p'})$ . □

LEMMA 2.3. *If  $m \in M_p^{cb}(G)$  the  $\tau_y m \in M_p^{cb}(G)$  and  $\|T_{\tau_y m}\|_{cb} = \|T_m\|_{cb}$ .*

*Proof.* Observe that

$$\left\| \sum_n e^{iy \cdot} f_n \otimes h_n \right\|_{L^p(S_p)}^p = \left\| \left( \sum_n f_n \otimes h_n \right) \right\|_{L^p(S_p)}^p$$

Hence,

$$\begin{aligned} & \left| \int_{\hat{G}} \left\langle \sum_n m(\xi - y) \hat{f}_n(\xi) h_n, \sum_n \hat{f}'_n(\xi) h'_n \right\rangle d\xi \right| \\ & = \left| \int_{\hat{G}} \left\langle \sum_n m(\xi) \widehat{e^{iy \cdot} f_n}(\xi) h_n, \sum_n \widehat{e^{iy \cdot} f'_n}(\xi) h'_n \right\rangle d\xi \right| \\ & \leq \|T_m\|_{cb} \left\| \sum_n e^{iy \cdot} f_n \otimes h_n \right\|_{L^p(S_p)}^p \left\| \sum_n e^{iy \cdot} f'_n \otimes h'_n \right\|_{L^{p'}(S_{p'})} \\ & \leq \|T_m\|_{cb} \left\| \sum_n f_n \otimes h_n \right\|_{L^p(S_p)}^p \left\| \sum_n f'_n \otimes h'_n \right\|_{L^{p'}(S_{p'})}. \end{aligned}$$

Hence by Lemma 2.2, we have  $\|T_{\tau_y m}\|_{cb} = \|T_m\|_{cb}$ . □

LEMMA 2.4. *Let  $k \in L^1(\hat{G})$  and  $m \in M_p^{cb}(G)$  then  $k * m \in M_p^{cb}(G)$  with  $\|T_{k * m}\|_{cb} \leq \|k\|_1 \|T_m\|_{cb}$ .*

*Proof.* We will use Lemma 2.2 to conclude that  $k * m \in M_p^{cb}(G)$ . Now for any finite sum we have

$$\begin{aligned} & \left| \int_{\hat{G}} \left\langle \sum_n k * m(\xi) \hat{f}_n(\xi) h_n, \sum_n \hat{f}'_n(\xi) h'_n \right\rangle d\xi \right| \\ & = \left| \int_{\hat{G}} \int_{\hat{G}} k(\eta) \left\langle \sum_n m(\xi - \eta) \hat{f}_n(\xi) h_n, \sum_n \hat{f}'_n(\xi) h'_n \right\rangle d\eta d\xi \right| \end{aligned}$$

$$\leq \|k\|_1 \|T_m\|_{cb} \left\| \sum_n f_n \otimes h_n \right\|_{L^p(S_p)} \left\| \sum_n f'_n \otimes h'_n \right\|_{L^{p'}(S_{p'})}.$$

The last inequality follows from Lemma 2.3. □

Proof of the following result follows from dominated convergence theorem and Lemma 2.2.

LEMMA 2.5. *Let  $\{m_n\} \subset M_p^{cb}(G)$  such that  $\lim_n m_n(x) = m(x)$  a.e. If  $\|T_{m_n}\|_{cb} \leq C < \infty$  then  $m \in M_p^{cb}(G)$ .*

*Proof of Theorem 1.3.* We will first prove it for  $m = \hat{k}$ , where  $k \in L^1(\mathbb{R})$ , and has compact support.

Define  $R : \mathbb{R} \rightarrow \mathcal{B}(L^p(\mathbb{T}))$  as  $R_x f(u) = f(u - x)$ , where  $x - u$  is interpreted as sum modulo 1. Taking  $S = R$ , the pair  $(R, S)$  satisfies all the hypotheses of Theorem 2.1. Hence, the operator  $H_k$  defined in Theorem 2.1 is a *cb* map. Observe that for  $f \in C(\mathbb{T})$ ,

$$\begin{aligned} \widehat{H_k f}(n) &= \int_{\mathbb{R}} k(x) \widehat{R_{-x} f}(n) dx \\ &= \int_{\mathbb{R}} k(x) e^{-2\pi i x n} \hat{f}(n) dx \\ &= m|_{\mathbb{Z}}(n) \hat{f}(n). \end{aligned}$$

We now prove the result for a general normalized multiplier  $m$ . Let  $\{\phi_n\}$  be as in Definition 1.2. Let  $m_n(x) = \hat{\phi}_n * m(x)$ . By Lemma 2.4 we have  $m_n \in M_p^{cb}(\mathbb{R})$ . Also,  $\lim_n m_n(x) = m(x) \forall x \in \mathbb{R}$ . As  $\|\hat{\phi}_n\|_1 = 1$  we have  $\|T_{m_n}\|_{cb} \leq \|T_m\|_{cb}$ .

Let  $\psi \in L^2(\mathbb{R})$  with  $\psi \geq 0$  with compact support and  $\int_{\mathbb{R}} \psi(x) dx = 1$ . We define  $h_n(x) = n\psi(nx)$ . Clearly  $\hat{h}_n(x) = \hat{\psi}(x/n) \rightarrow 1$  as  $n \rightarrow \infty$ . Consider the sequence  $\tilde{k}_n = (m_n \hat{h}_n)^\vee$ . Since  $m_n \hat{h}_n \in L^2(\mathbb{R})$  and their Fourier transform has compact support we conclude  $\tilde{k}_n \in L^1(\mathbb{R})$  with compact support. Hence, from our earlier observation  $\hat{\tilde{k}}_n|_{\mathbb{Z}} \in M_p^{cb}(\mathbb{T})$  with uniform *cb* norm. Now  $\hat{\tilde{k}}_n(x) \rightarrow m(x), \forall x \in \mathbb{R}$  as  $n \rightarrow \infty$ . Finally, by Lemma 2.5 we have  $m|_{\mathbb{Z}} \in M_p^{cb}(\mathbb{T})$ . □

### 3. *cb* homomorphism theorem for locally compact Abelian group

In this section, we address the problem of strict inclusion  $M_p^{cb}(G) \subsetneq M_p(G)$  for non-compact locally compact Abelian group  $G$ . We will achieve this by transferring the known result on compact Abelian group to this set up. As in Section 2 the main ingredient here is the following *cb* version of multiplier homomorphism theorem [6], [1].

**THEOREM 3.1.** *Let  $G_1$  and  $G_2$  be two locally compact Abelian groups. Suppose  $m \in M_p^{cb}(G_1)$  and is continuous. If  $\pi : \hat{G}_2 \rightarrow \hat{G}_1$  be a continuous homomorphism, then  $m \circ \pi \in M_p^{cb}(G_2)$ . Moreover,*

$$\|m \circ \pi\|_{cb} \leq \|m\|_{cb}.$$

Suppose  $\pi : \hat{G}_2 \rightarrow \hat{G}_1$  is a continuous homomorphism. Observe that such a  $\pi$  will induce a continuous homomorphism  $\hat{\pi}$  from  $G_1$  to  $G_2$  namely,  $\hat{\pi}(x)(\gamma) = \pi(\gamma)(x) \forall x \in G_1, \gamma \in G_2$ . Now for  $f \in L^p(G_2)$  we define  $R : G_1 \rightarrow \mathcal{B}(L^p(G_2))$  by  $R_u f(x) = f(\hat{\pi}(u)x)$ . Taking  $R = S$  the pair  $(R, S)$  satisfies the conditions of Theorem 2.1. Arguing exactly in the same fashion as in Theorem 1.3 we have Theorem 3.1 if  $m = \hat{k}$  for some  $k \in L^1(G_1)$  having compact support. We can get the result for general  $m$  if there exists an approximate identity  $\{\phi_i\}$  in  $L^1(G_1)$  satisfying following conditions:

- (1)  $\phi_i \geq 0$  and  $\int_{G_1} \phi_i(x) dx = 1 \forall i \in I$ .
- (2)  $\hat{\phi}_i \in C_c(\hat{G}_1) \forall i \in I$ .
- (3)  $\lim_i \hat{\phi}_i(\gamma) = 1$  and the convergence is uniform on all compact subsets of  $\hat{G}_1$ .

Existence of such an approximate identity  $\{\phi_i\}$  is guaranteed in [7, Theorem 33.12].

Now we consider our problem of constructing an  $L^p(G)$  multiplier which is not a  $cb$  multiplier. Consider a discrete subgroup  $K$  of  $\hat{G}$ . Then  $H = \hat{K}$  is compact Abelian. Therefore, we can get a  $L^p(H)$  multiplier  $\phi$  which is not  $cb$ . By a construction of Cowling [4], we can extend  $\phi$  continuously to a function  $\psi \in M_p(G)$  such that  $\psi|_K = \phi$ . If  $\psi \in M_p^{cb}(G)$ , then by Theorem 3.1 we have  $\phi \in M_p^{cb}(H)$  which is a contradiction. Hence, for every locally compact Abelian group  $G$ , the inclusion  $M_p^{cb} \subsetneq M_p(G)$  is strict.

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